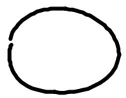
AIR RESISTANCE w/ calc.

Terminal velocity is when a goes to zero b/c $F_g = F_r$

$$\vec{F}_r = -b\vec{v}$$

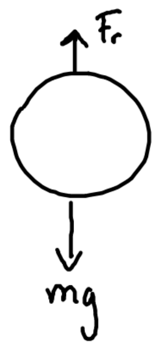
$$= -b\vec{v}^2 \quad \text{"easy"}$$



$V_0 = 0$ drop the ball

$F_r = -kv$

find $v_{terminal}$.



1) FBD

2) $\Sigma F = ma$

$mg - F_r = m\cancel{a}^{\,0}$ b/c $V_{term.}$

$mg = F_r$

$mg = kv$

$\frac{mg}{k} = V_{term.}$

we need to find $v(t)$ for any t

From FBD we know $mg - kv = ma$

$$\frac{mg-kv}{m} = a \qquad a = \frac{dv}{dt}$$

$$\frac{mg-kv}{m} = \frac{dv}{dt}$$

to solve for v, isolate V with dv

$$\frac{1}{m} = \frac{1}{mg-kv}\frac{dv}{dt}$$

move dt to other side

$$\int \frac{1}{m}dt = \int \frac{1}{mg-kv}\,dv$$

integrate both sides

....

power rule does not work for

$$\int \frac{1}{x}dx \quad b/c \quad \int x^{-1}dx = \frac{x^0}{0}$$

OH NO !!

$$\boxed{\int \frac{1}{x}dx = \ln|x| + C}$$

$\int \frac{1}{m}\, dt = \int \frac{1}{mg-bv}\, dv$ use u sub

coefficient

let $u = mg - bv$ constant

$\left(\frac{1}{m}\right)t + C = \int \frac{1}{u}\, dv$ $du = -b\, dv$

$\frac{t}{m} + c = \int \frac{1}{u}\left(\frac{du}{-b}\right)$ $\frac{du}{-b} = dv$

replace dv w/

$\frac{t}{m} + c = -\frac{1}{b}\int \frac{1}{u}\, du$

$\frac{t}{m} + c_1 = -\frac{1}{b}\ln|u| + c_2$

$\frac{t}{m} + C = -\frac{1}{b}\ln|mg-bv|$ isolate v

$-\frac{bt}{m} + C = \ln|mg-bv|$

$e^{-\frac{bt}{m}+C} = e^{\ln|mg-bv|}$

$e^{-\frac{bt}{m}} \cdot e^{C} = mg - bv$

$Ce^{-\frac{bt}{m}} = mg - bv$

$Ce^{-\frac{bt}{m}} - mg = -bv$

$\frac{Ce^{-\frac{bt}{m}} - mg}{-b} = v$

$\frac{mg}{b} - Ce^{-\frac{bt}{m}} = v$

Solve for C

plug in knowns

v_0, t_0

$v_0 = 0$ @ $t = 0$

$\frac{mg}{b} - Ce^{-\frac{b(0)}{m}} = 0$

$-Ce^{0} = -\frac{mg}{b}$

$C = \frac{mg}{b}$ plug back in for C

$\frac{mg}{b} - \frac{mg}{b}e^{-\frac{bt}{m}} = v$

$\frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right) = v$ done!!