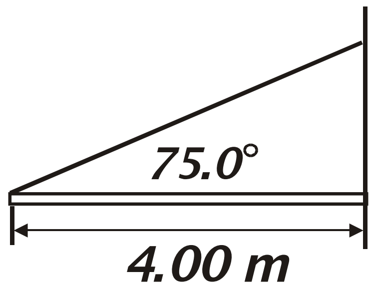
AP Physics – Applying Torque

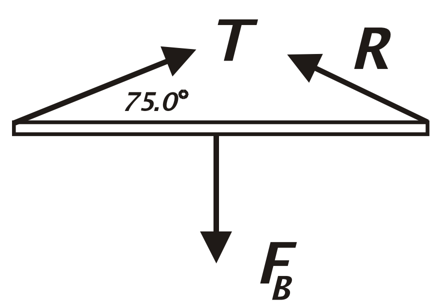
It is now time to go after some problems that are more complicated. You will find these to be a lot of phun. Honest.

- A uniform beam is supported by a stout piece of line as shown. The beam weighs 175 N. The cable makes an angle of 75.0° as shown. Find (a) the tension in the cable and (b) the force exerted on the end of the beam by the wall.



We can solve this problem by summing forces and adding up torques.

First let's draw a FBD:



We have three forces acting on the beam.

The weight of the beam which acts at the center of the beam (its CG), $\boldsymbol{F_B}$.

The tension in the cable, $\boldsymbol{T}$.

And the force exerted by the wall on the beam, $\boldsymbol{R}$. (The wall is pushing the beam up and out.)

(a) Let us first sum the torques. The pivot point is the end of beam where it meets the wall. Therfore $\boldsymbol{R}$ exerts no torque as its lever arm is zero. We only have two torques to deal with and, of course, they add up to zero. Torque one is exerted by the tension in the cable and torque two is caused by the weight of the beam. The force for this torque is applied at the CG, which is at the center of the beam. Only the vertical component of the tension causes its torque so:

$$\tau_{cable} - \tau_B = 0 \qquad Tr\sin\theta - F_B\left(\frac{r}{2}\right) = 0$$

$$T = \frac{F_B \cancel{r}}{2\cancel{r}\sin\theta} = \frac{175\ N}{2\sin 75.0^{\circ}} = \boxed{90.6\ N}$$

(b) Next we can sum up the forces:

x direction: $T\cos\theta - R_X = 0$ ***y*** direction: $T\sin\theta - F_B + R_Y = 0$

We can solve the ***x*** direction equation for $\boldsymbol{R_X}$:

$$R_X = T\cos\theta \quad = (90.6\ N)\cos 75.0^\circ \quad = \quad 23.4\ N$$

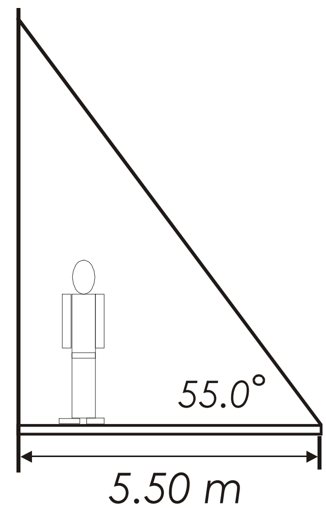
Next we solve for $\boldsymbol{R_Y}$:

$$R_Y = F_B - T\sin\theta \quad = 175\ N - (90.6\ N)\sin 75.0^\circ \quad = \quad 87.5\ N$$

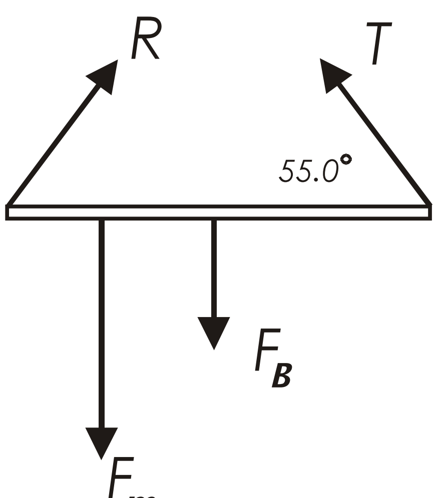
We've found the x and y components for *R*, so now we can find the magnitude of the vector using the Pythagorean theorem:

$$R = \sqrt{{R_y}^2 + {R_x}^2} \quad = \sqrt{(87.5\ N)^2 + (23.4\ N)^2} \quad = \quad \boxed{90.6\ N}$$

- A beam is supported as shown. The beam is uniform and weighs 300.0 N and is 5.00 m long. A 635 N person stands 1.50 m from the building. (a) What is the tension in the cable and (b) the force exerted on the beam by the building?



We draw a FBD.



(a) Sum of torques:

$$\tau_{beam} + \tau_{man} - \tau_{cable} = 0$$

$$F_m r_m + F_B r_B - T r_C \sin\theta = 0 \qquad T = \frac{F_m r_m + F_B r_B}{(\sin\theta) r_C}$$

$$T = \frac{635\ N(1.50\ \cancel{m}) + (300.0\ N)(2.50\ \cancel{m})}{(\sin 55.0^\circ)5.00\ \cancel{m}} \quad = \quad \boxed{416\ N}$$

(b) We can resolve ***R*** and ***T*** into components and then sum the forces in the ***x*** and ***y*** direction. All forces must add up to equal zero.

$$R_X - T\cos\theta = 0 \qquad R_y + T\sin\theta - F_B - F_m = 0$$

$$R_X - T\cos\theta = 0 \qquad R_X = T\cos\theta = (416\ N)\cos 55.0^0 = 238\ N$$

$$R_y + T\sin\theta - F_B - F_m = 0 \qquad R_y = F_B + F_m - T\sin\theta$$

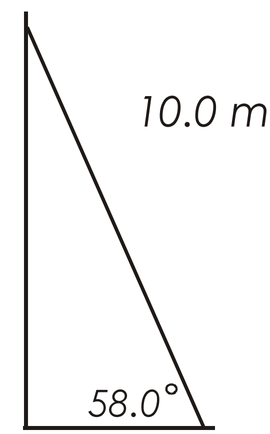
$$R_y = 300.0\ N + 635\ N - (416\ N)\sin\theta = 624\ N$$

$$R = \sqrt{R_y^{\,2} + R_x^{\,2}} = \sqrt{(624\ N)^2 + (238\ N)^2} = \boxed{668\ N}$$

Fabulous Ladder Problems: Ladder problems are very popular. The basic idea is that you have a ladder leaning against a wall (which is usually frictionless). The ladder is held in place by the friction between its base and the deck it rests upon. We're given the situation and then required to figure out various things – the angle the ladder makes with the deck, the friction force, the coefficient of friction, the force exerted on the top of the ladder by the wall, &tc.

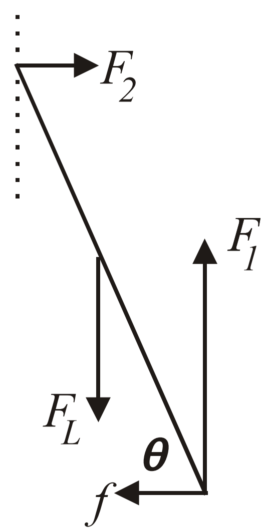
Let's go ahead and do a simple problem.

- A uniform 250.0 N ladder that is 10.0 m long rests against a frictionless wall at an angle of 58.0°, the ladder just keeps from slipping. (a) What are the forces acting on the bottom of the ladder? (b) What is the coefficient of friction of the bottom of the ladder with the ground?



Draw a FBD.

The forces acting on the ladder are: the weight of the ladder $\boldsymbol{F_L}$, the frictional force $\boldsymbol{f}$, The force the deck pushes up on the ladder with $\boldsymbol{F_1}$, and the force exerted by the wall on the top of the ladder $\boldsymbol{F_2}$.



Now we look at the forces acting on ladder - they have to add up to zero.

$$\sum F_y = 0 \qquad F_1 - F_L = 0 \qquad F_1 = F_L = \boxed{250\ N}$$

$$\sum F_x = 0 \qquad f - F_2 = 0 \qquad f = F_2$$

We have to find either $\boldsymbol{F_2}$ or else $\boldsymbol{f}$. But we need more info, don't we? You bet we do. Blessed by good fortune as we are, we instantly recognize that we can make use of the torque equilibrium deal.

First we make a drawing showing all the torques acting on the ladder. (Actually we're only looking at the forces that are perpendicular to the lever arm.)

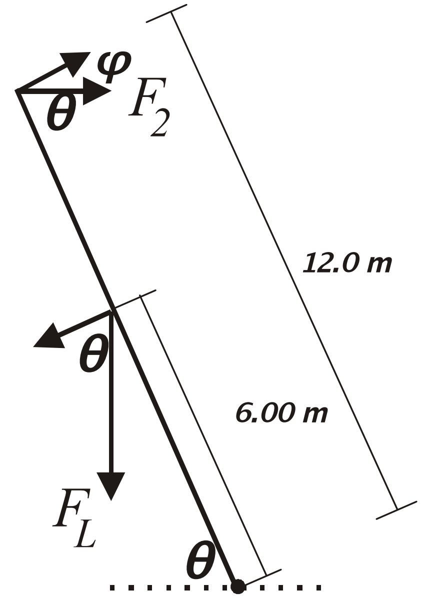
The pivot point is the base of the ladder.

Neither the friction or F_1 cause a torque as their lever arm is zero.

The weight of the ladder causes a CCW torque.

The lever arm from F_2 causes a CW torque.

The torques add up to zero.



$$\sum \tau = 0$$

$$\tau_{ladder} + \tau_{wall} = 0$$

The angle ϕ is, using geometry clearly going to be:

$$\phi = 90^o - \theta \qquad = 90^o - 58^o \quad = \quad 32^o$$

The torques are: $F_2 \cos\phi \, d_2 - F_L \cos\theta \, d_1 = 0$ Solve for F_2:

$$F_2 = \frac{F_L \cos\theta \, d_1}{\cos\phi \, d_2} = \frac{250.0 \, N \cos 58.0^\circ \left(6.00 \, \cancel{m}\right)}{\cos 32.0^\circ \left(12.0 \, \cancel{m}\right)} = 78.1 \, N$$

The frictional force (the other force acting at the base of the ladder is therefore:

$$f = \boxed{78.1 \, N}$$

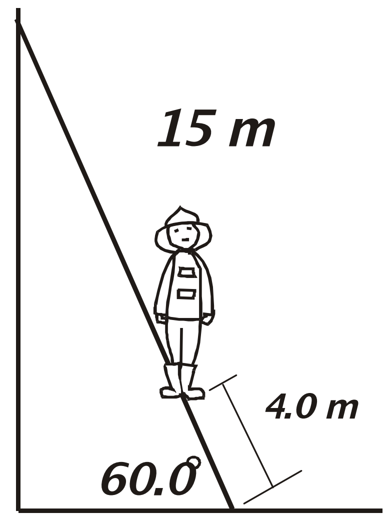
(b) Find the coefficient of friction:

$$f = \mu n \qquad \mu = \frac{f}{n} \qquad \mu = \frac{78.1 \, \cancel{N}}{250.0 \, \cancel{N}} = \boxed{0.312}$$

Whew! Can we make it worse? You bet.

The longest recorded flight of a domestic chicken is 13 seconds.

- A 15 m, 500.0 N uniform ladder rests against a frictionless wall. It makes 60.0° angle with the horizontal. Find (a) the horizontal and vertical forces on the base of the ladder if an 800.0 N fire fighter is standing 4.0 m from the bottom. If the ladder is on the verge of slipping when the fire fighter is 9.0 m from the bottom of the ladder, (b) what is the coefficient of static friction on the bottom?



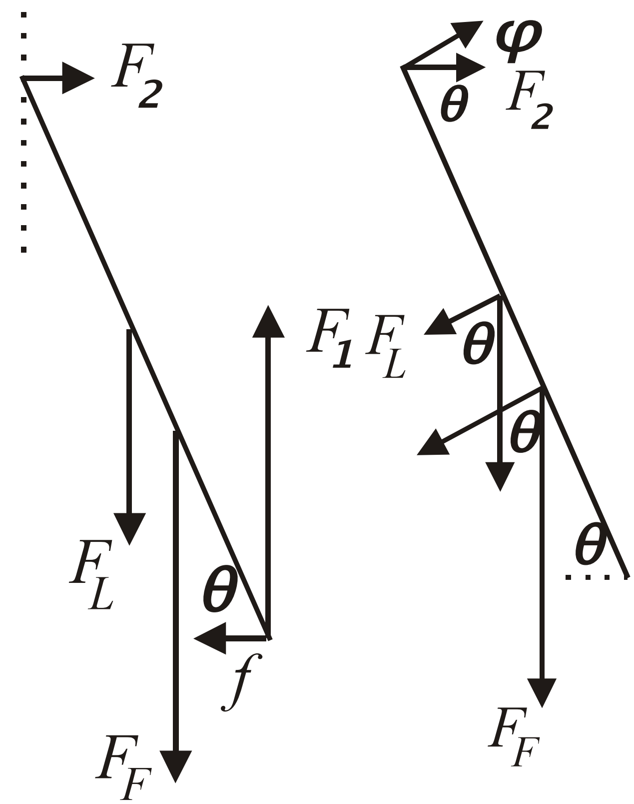
$$\sum F_Y = 0$$

$$F_1 - F_L - F_F = 0$$

$$F_1 = F_L + F_F$$

$$F_1 = 500\ N + 800\ N$$

$$F_1 = 1300\ N$$



$$\sum F_X = 0 \qquad F_2 - f = 0$$

$$f = F_2$$

Let's look at torque to find $\boldsymbol{F_2}$:

$\sum \tau = 0$ Pivet point is at the base of the ladder:

$$\phi = 90^\circ - \theta = 90^\circ - 60^\circ = 30^\circ$$

$$\tau_2 - \tau_L - \tau_F = 0$$

$$F_2 \cos\phi\, r_2 - F_L \cos\theta\, r_L - F_F r_F = 0$$

$$F_2 = \frac{F_L \cos\theta\, r_L + F_F r_F}{\cos\phi\, r_2}$$

$$F_2 = \frac{500.0\ N \cos 60.0^\circ\,(7.50\ \cancel{m}) + 800.0\ N \cos 60.0^\circ\,(4.00\ \cancel{m})}{\cos 30.0^\circ\,(15.0\ \cancel{m})}$$

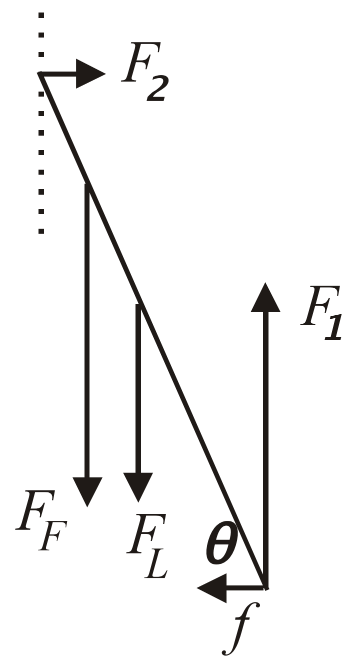
$$F_2 = 270\ N \qquad so \qquad f = 270\ N$$

So the force up is $F_1 =$ $\boxed{1300\ N}$

The horizontal frictional force is: $f =$ $\boxed{270\ N}$

(b) When the fire man is at 9.0 m (we'll figure this from the bottom of the ladder), then

We can use the same equation as we used to find F_2 since the only thing that has changed is the distance of the firefighter from the bottom of the ladder.



$$F_2 = \frac{F_L \cos\theta\, r_L + F_F r_F}{\cos\phi\, r_2}$$

$$F_2 = \frac{500.0\ N \cos 60.0^\circ\,(7.50\ m) + 800.0\ N \cos 60.0^\circ\,(9.00\ m)}{\cos 30.0^\circ\,(15.0\ m)}$$

$$f = 421\ N$$

We can now find the coefficient of static friction for the bottom of the ladder and the deck.

$$f = \mu_s n \qquad \mu_s = \frac{f}{n} = \frac{421\ N}{1300\ N} = \boxed{0.32}$$

All there is to it.