AP Physics – Momentum

Momentum is a common word – you hear it every so often, but not like every day, normally. Sportscasters will say that a team has a "lot of momentum". Or a news anchor person might say "the need for federal regulation of the qualification of physics teachers is gaining momentum". A kind of related thing would be the idea that something is momentous (which would mean that it was important).

Momentum sounds important too. If something has momentum, well, that's got to be a big deal. It has ***MOMENTUM***!

Well, forget all that! In physics momentum is simply the velocity of an object multiplied by its mass.

When something is at rest it has a certain quality which is very different from the one it has when it is moving. You would feel safe stepping in front of a locomotive and pushing on its nose – if it were at rest. But you would not want to do this if it was moving. Especially if it was moving fast. This is because of its momentum.

Momentum can be thought of as sort of like inertia in motion. Recall that inertia is the property of matter that is responsible for Newton's first law. It resists changes in a system's motion. Anyway, the thing with momentum is that when things are moving, the inertia seems to get amplified or something. See, when something gets underway, the ***faster*** it ***goes*** the more it seems to want to ***keep*** moving.

Here's an important thing about momentum:

Momentum is a vector quantity.

Momentum in mathematical terms is:

$$p = mv$$ ***Momentum Equation***

Where ***p*** is momentum, ***m*** is mass, and ***v*** is velocity.

Momentum has units of: $\frac{kg \cdot m}{s}$ or $N \cdot s$

To have momentum, an object must have a velocity - be moving. A strand of spider web silk drifting on the air currents has more momentum than a mile long coal train that is waiting on a siding.

- What is the momentum possessed by a car with a mass of 2 350 kg if it is traveling at 125 km/h?

The velocity must be expressed in units of m/s, so we do the conversion:

$$125\frac{\cancel{km}}{\cancel{h}}\left(\frac{1\ \cancel{h}}{3\,600\ s}\right)\left(\frac{1\,000\ m}{1\ \cancel{km}}\right) = 34.72\text{ m/s}$$

$$p = mv \quad = 2\,350\ kg\left(34.72\frac{m}{s}\right) \quad = \quad \boxed{8\,160\ \frac{kg\cdot m}{s}}$$

Because momentum is a vector, we can always break it down into its horizontal and vertical components:

$$p_x = mv_x \qquad\qquad p_y = mv_y$$

Impulse: Today, we use ***F*** = ***ma*** as a quick definition of the second Law of motion. But Newton did not express it in that form. Instead he said that a force was the rate of change of momentum. Instead of ***F*** = ***ma*** he said:

$$F = \frac{\text{change in momentum}}{\text{time interval}} \qquad \text{or} \qquad F = \frac{\Delta p}{\Delta t}$$

It turns out that this is the same as F = ma – if you do some algebra type fiddling around.

$$F = \frac{\Delta p}{\Delta t} = \frac{mv - mv_o}{\Delta t} = \frac{m\left(v - v_o\right)}{\Delta t}$$

Recall that $v = v_o + at$ We can plug this into the equation we developed:

So $$F = \frac{m\left(v - v_o\right)}{\Delta t} - \frac{m\left(\left(v_o + at\right) - v_o\right)}{\Delta t} \qquad F = \frac{m\left(\cancel{v_o} + a\cancel{t} - \cancel{v_o}\right)}{\cancel{t}} = ma$$

Thus: $F = ma$

Now let's go back to Newton's original equation:

$F = \dfrac{\Delta p}{\Delta t}$ Multiply both sides by ***Δt***: $F\Delta t = \Delta p$

Which is: $F\Delta t = m\left(v - v_o\right)$

The quantity on the left, ***FΔt***, is called ***impulse***.

$$F\Delta t = Impulse$$

The AP Test has this equation which is used for impulse:

$$J = F\Delta t = \Delta p$$

Here $\boldsymbol{J}$ is the impulse, $F\Delta T$, and Δp is the change in momentum.

The impulse is the applied force multiplied by the time over which it acts. This quantity is equal to the change in momentum of a system.

We can say that impulse changes momentum.

We can look at two events, both involving a car, a 1 200 kg car. We've got this car that is traveling along at 75 mph on the interstate. That's about 125 km/h or roughly 35 m/s. We want to stop the car. This means that the car will undergo a change in momentum of:

$$\Delta p = m\Delta v = 1\,200\ kg\left(35\frac{m}{s}\right) \quad = \quad 42\,000\frac{kg\cdot m}{s}$$

This will be the change in momentum no matter what. Now we'll bring in the impulse, ***J*** recall that it is really (***F*Δ*t***). If we use the brakes to come to a nice, controlled stop, it will take a fairly long time to stop the car – maybe 12 seconds. But if we run into a humungous, massive boulder that has rolled down the mountain onto the roadway, the car will come to a stop in an extremely short amount of time. Less than a second.

In both cases the car will have the same change in momentum. It will also undergo the same impulse. The difference will be in the time and force that it takes to stop the car.

Using the brakes, the force acting on the car will be small. Using the boulder, the force that acts on the car will be enormous. Can you see why this is so?

Let's use some numbers with our car example.

Case 1: Using the brakes: $J = F\Delta t = \Delta p \quad F = \dfrac{\Delta p}{\Delta t} \quad = \dfrac{m\left(v - v_o\right)}{\Delta t}$

$$F = 42\,000\frac{kg\cdot m}{s}\left(\frac{1}{12.0\ s}\right) \quad = \quad \boxed{3\,500\ N}$$

Case 2: Hitting the boulder:

$$F = 42\,000\frac{kg\cdot m}{s}\left(\frac{1}{0.010\ s}\right) \quad = \quad \boxed{4\,200\,000\ N}$$

Stopping the car with brakes requires 3 500 N, stopping the car with the boulder has a force of over four million newtons! Thee force is about ***12 000 time bigger!*** This is why stopping a car with brakes is a safe, rational thing to do, but running into a boulder to effect the same velocity change is crazy.

So, if we apply a large force for a short time we can generate a momentum change of the same magnitude as having a small force for a long time.

$$\Huge F\,\small t \quad = \quad \small F\,\Huge t$$

- What impulse is required to stop a 0.25 kg baseball traveling at 52 m/s? (b) If the ball is in the fielder's mitt for 0.10 seconds as it is being stopped, what average force was exerted on the ball?

(a) $J = F\Delta t = \Delta p \quad J = m\left(v - v_o\right) \quad = 0.25\,kg\left(0 - \left(-52\,\frac{m}{s}\right)\right) \quad = \quad \boxed{13\,kg\frac{m}{s}}$

(b) Now we can find the force: $F\Delta t = \Delta p \qquad F = \dfrac{\Delta p}{\Delta t}$

$$F = 13\,kg\frac{m}{s}\left(\frac{1}{0.10\,s}\right) \quad = \quad \boxed{130\,N}$$