AP Physics - Free Fall

Aristotle (384 – 322 BC), one of your basic ancient Greek philosophers, said that things fall because they want to regain their natural state - earth with earth, water with water, and so on. Thus a rock will fall back to the earth to be with the other rocks. Since a big rock possesses more "earth", it will fall faster than would, say, a feather (which is woefully inadequate in the earth amount category compared with your basic rock). Aristotle's idea appears to be true because a rock certainly falls faster than a feather. In fact it made so much sense, that Aristotle's ideas on the subject were the accepted truth for around 2 000 years until the Renaissance.

The first scientific study of gravity was done by Galileo Galilei (1564 - 1642). He was trained as a mathematician and was a university professor. In the late 1500's Galileo conducted a series of experiments on gravity. He is supposed to have demonstrated that heavy objects and light objects fall at the same speed. The act of doing experiments to find out what would happen – this was a very daring idea.

Here is Galileo's account of the experiment from his book, *Dialogues of two New Sciences*.

> "But I, Simplicio, who have made the test can assure you that a cannon ball weighing one or two hundred pounds or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits...the larger outstrips the smaller by two finger-breadths, that is, when the larger has reached the ground, the other is short of it by two finger-breadths.

Galileo did not, as is popularly believed, state that the objects would hit the ground at the same time – he understood air resistance. He did understand that without air resistance, the objects ***would*** fall at exactly the same rate.

Galileo wrote about doing the experiment as if he had done it several times, but it is not clear where or when he did it. The story that he dropped cannon balls from the Leaning Tower of Pisa has only one source, his last pupil and biographer, Vincenzo Vivani. He describes a very public event -- the entire university in attendance to witness the thing. But no one at the university ever mentioned witnessing the event. So whether Galileo did or did not do the experiment is sort of up in the air.

Galileo's idea that things fall at the same rate flies in the face of common sense. It seems reasonable that heavy things ought to fall faster than light ones

To study gravity, Galileo found that he had to slow it down. This was because he couldn't measure the time it took an object to fall with the crude instruments of the time. Gravity was "slowed down" by having balls roll down inclined planes (ramps). Gravity still caused the motion, but its effect was decreased to the point where Galileo could gather useful data. Galileo found that the distance that accelerated objects would travel was proportional to the square of the time. More on this later.

Acceleration of Gravity: On the earth, gravity exerts a force on everything with mass. (A ***force*** is a push or pull.) The force makes all objects accelerate downwards, towards the center of the earth. This acceleration varies a tiny little bit depending on where you are - at the North Pole this acceleration is 9.83217 m/s^2 and at the Equator it has a value of 9.78039 m/s^2. This is because the earth is not a perfect sphere. Fortunately we can safely ignore the tiny differences in the acceleration of gravity. The value which is commonly used for this acceleration is 9.80 m/s^2. In English units it is 32.0 ft/s^2. Gravity's acceleration is kind of special so it is given its very own little symbol, ***g***.

$$g = 9.80 \frac{m}{s^2}$$

Drop a rock from the top of a cliff and, in one second, it will reach a speed of 9.80 m/s, after two seconds it will be traveling at 19.6 m/s, in three seconds it's going 29.4 m/s, at four seconds it's speed will be up to 39.2 m/s, and so on. It looks like the rock will keep going faster and faster and faster until it smashes into the earth, and it would, if it were falling in a vacuum. The thing is, see, that the air causes a frictional force that opposes the rock's fall and slows it down. For short drops with dense objects (like rocks) we can reasonably ignore the effects of the air. Oh, the fancy, scientific term for this force exerted by the air is ***drag*** or ***air resistance***, sometimes it is called ***wind resistance***. At high velocities or over long distances, the drag can become significant, especially for objects that are not dense, like feathers or leaves or fluff. In the real world, an object in free fall will accelerate to its ***terminal velocity***. This is the speed at which the force of gravity equals the drag force. The object then stops accelerating and falls at a constant velocity. People jumping out of airplanes experience this. The typical laid out position that sky divers use gives them a terminal velocity of around 100 mph.

When an object is released and allowed to fall, its motion can be described by the following table (ignoring air resistance):

Time	Velocity	Distance
1 s	9.8 m/s	4.6 m
2 s	19.6 m/s	19.6 m
3 s	29.4 m/s	44.1 m
4s	39.2 m/s	78.4 m
5s	49.0 m/s	122 m
&tc		

The kinematic acceleration equations can be used to describe the motion of falling objects.

- A ball is thrown straight upward. If it takes 4.25 seconds to reach the top of its path, what is its initial speed?

Since the ball is traveling upward, and the acceleration is downward, the ball will slow down as it moves up. For the upward part of its motion, its final velocity will be zero – it will then momentarily come to rest and then change direction and begin to accelerate downward. Since we know that for the upward part of its journey the final velocity is zero, we can easily calculate the initial velocity.

$$\cancel{v} = v_0 + at \quad v_0 = -at \quad = -\left(-9.80\frac{m}{s^{\cancel{2}}}\right)\left(4.25\,\cancel{s}\right) = \boxed{41.6\frac{m}{s}}$$

The velocity and acceleration have opposite directions, so one of the quantities must be negative. We've chosen "down" as the negative direction for the above solution (but hey, you could choose up to be negative if you like).

- A stone is thrown straight up from top of building with an initial speed of 35.5 m/s. (a) How high does it go from the top of the building? (b) How much time to reach the maximum height? (c) If the building is 45.2 m tall, how much time will it take to hit the ground from when it was initially launched?

(a) $v^2 = v_o^2 + 2ay$ $\quad$ v at top is zero so; $\quad$ $0 = v_o^2 + 2ay$

$$2ay = -v_o^2 \qquad y = \frac{-v_o^2}{2a} = -\left(35.5\frac{m}{\cancel{s}}\right)^2\left(\frac{1}{2\left(-9.80\frac{\cancel{m}}{\cancel{s}^2}\right)}\right) = \boxed{64.3\ m}$$

(b) $v = v_o + at \qquad 0 = v_o + at \qquad t = \frac{-v_o}{a}$

$$t = -35.5\frac{\cancel{m}}{\cancel{s}}\left(\frac{1}{-9.80\frac{\cancel{m}}{s^{\cancel{2}}}}\right) = \boxed{3.62\ s}$$

(c) ***The stone takes 3.62 s to reach the highest point in its path, it then must fall 64.3 m (to the top of the building) and then another 45.2 m to hit the deck below. So figure the problem from the top of the ball's path, where its velocity is zero and just before it begins to fall back down.***

It's initial velocity is zero, and, since the stone will be falling down, we can, what the heck, assume that down is positive (we can do this! We are in charge!):

$$y = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2y}{a}} \qquad t = \sqrt{\frac{2\left(64.3\,\cancel{m} + 45.2\,\cancel{m}\right)}{9.80\frac{\cancel{m}}{s^2}}} = 4.73\ s$$

The total time for the ball to be in the air is $\quad 3.62\ s + 4.73\ s = \boxed{8.35\ s}$

Negative or Positive: You get to select the coordinate system that you use to solve problems. This means you get to decide where the displacement is zero and what direction will be positive or negative. Look at what happens if you have a negative acceleration, such as **– 9.8** $\boldsymbol{m/s^2}$. Does this mean the object is decelerating (slowing) or does it mean that the object is moving along a negative (perhaps the ***y***) axis? It would depend on the problem. For an object moving on the ***x***-axis it would mean decelerating. For an object falling along the ***y***-axis, due to gravity, it means the object is accelerating, but in the downward direction. You choses you your directions for this stuff basically so that the calculations and everything are easiest.