

AP Physics – Vectors 101

Good Stuff on Vectors: We’ve briefly mentioned vectors and scalars. The time has now come to undertake a thorough study of the things.

Of the two, scalars are the easiest to understand and deal with. See, when you deal with scalars, it’s all very simple, they’re just like everyday numbers -- just use the regular arithmetic stuff that you have laboriously mastered during the past 10 – 12 years. Like adding apples with apples.

Dealing with vectors is a whole ‘nuther thang’. It’s more like adding apples and pork bellies together or something.

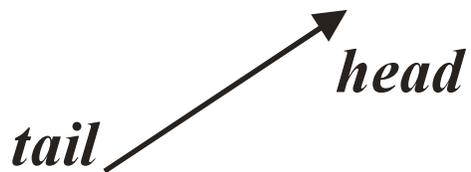
When you add scalars you simply do this: $2 \text{ kg} + 3 \text{ kg} = 5 \text{ kg}$.

But with vectors, $2 + 3$ might equal 4! Lord, how can this be!

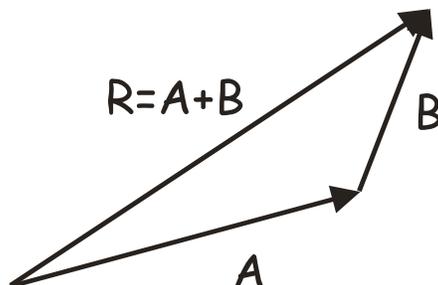
Well, your basic vectors have both magnitude and direction. If you add two velocities, one that is north at 10 m/s and the other is south at 10 m/s, you end up with zero m/s. Can you see why this is so?

The classic way to add vectors is to represent them with arrows, draw them to scale, and then measure out things with a ruler and protractor to see what the answer would be.

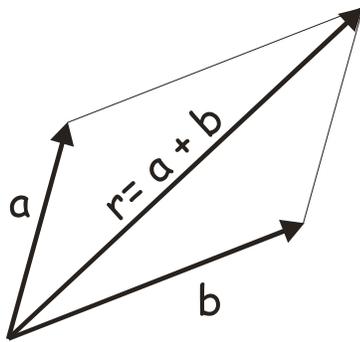
A vector is represented as an arrow. The arrowhead points in the vector’s direction and the length represents the vector’s magnitude. To represent the magnitude, you have to use a scale to figure out how long to draw the thing. Each vector has a head and a tail.



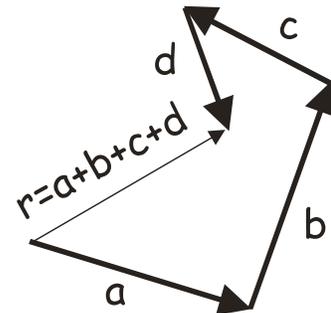
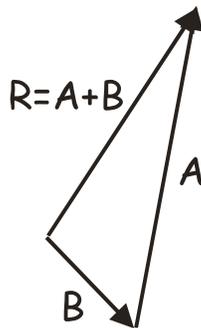
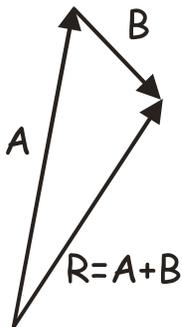
The graphical vector addition method is sometimes called the “head-to-tail” method. You draw one vector, then draw the second vector with its tail on the head of the first. The next vector would be drawn with its tail on the head of the second and so on. The final answer, known as the **resultant vector**, is found by drawing a vector from the tail of the first vector to the head of the last.



It makes no difference in what order you add the vectors as you can see by the drawing below.



You will not be required to add vectors graphically – it’s a lot of trouble and not very accurate and there are better ways to do it. Here are some other examples.



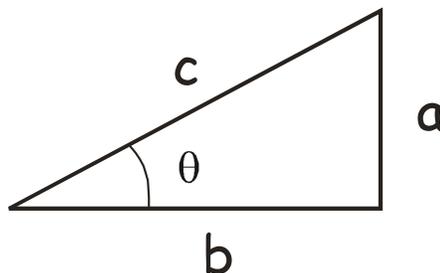
So how do we go about doing this? Well, fudidaboutit. It’s way too much trouble. You need to measure things super accurately, draw angles precisely, &tc. This is all very tedious stuff. It’s far easier to add them utilizing other, better methods. Thus we will not use the head to tail thing.

Here’s what you should be able to do with the head to tail bit: you *should* be able to make a rough drawing of the vector addition. This is very useful because it will give you an idea of what the approximate direction and magnitude of the resultant vector ought to look like.

Trigonometry: So how do we solve vector problems? Well, gosh, thanks for asking. Here’s how we do the thing - we use trigonometry!

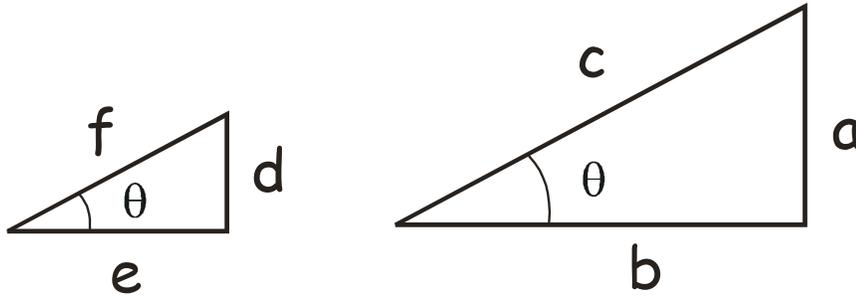
Actually, you’re going to get to use a heck of a lot of trigonometry in this course, you lucky devil.

Here is a *right triangle*. A right triangle has a ninety-degree angle (called a *right angle*) in it.



In reference to angle θ , side a is called the opposite side (it is farthest away from the angle), side b is called the adjacent side (it is “adjacent” to the angle, i.e., right next to it), and side c is called the hypotenuse.

Any two right triangles that have the same interior angle θ are similar and the ratios of the sides are a constant (you will, it is to be hoped, recall this from your old geometry class). Observe the drawing below:



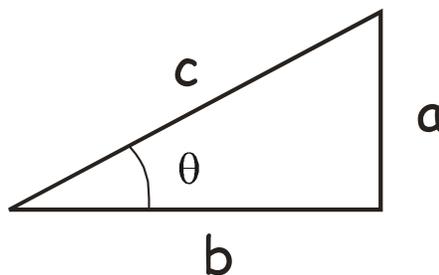
The ratios for the sides of the two triangles is:

$$\frac{e}{f} = \frac{b}{c} \quad \frac{d}{f} = \frac{a}{c} \quad \frac{d}{e} = \frac{a}{b}$$

This is what trigonometry is based upon – the ratio of the sides of similar triangles. We will be using three trigonometry functions, that is, three ratios; the sine, cosine, and tangent.

These functions are defined as:

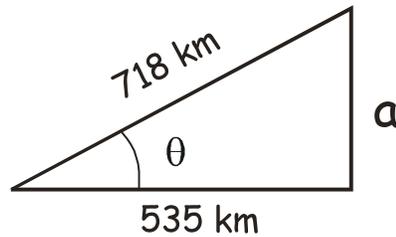
$$\sin \theta = \frac{\text{side opposite} \theta}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{side adjacent} \theta}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{side opposite} \theta}{\text{side adjacent} \theta}$$



For our original triangle:

$$\sin \theta = \frac{a}{c} \quad \cos \theta = \frac{b}{c} \quad \tan \theta = \frac{a}{b}$$

- A plane travels east 535 km, then flies north, then flies back to its takeoff point a distance of 718 km. (a) How far north did it travel? (b) What is the angle θ ?



(a) A simple way to find the distance the plane traveled north would be to use the Pythagorean theorem.

$$c^2 = a^2 + b^2 \quad \text{solve for a:}$$

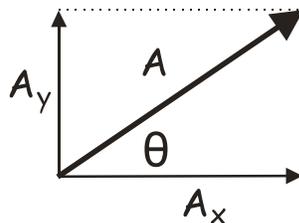
$$a = \sqrt{c^2 - b^2} = \sqrt{(718 \text{ km})^2 - (535 \text{ km})^2} = \boxed{479 \text{ km}}$$

(b) To find the angle we can use the tangent function (opposite side over adjacent side):

$$\tan \theta = \frac{a}{b} = \frac{479 \text{ km}}{535 \text{ km}} \quad \theta = \boxed{41.8^\circ}$$

This is actually a vector problem – it may be your very first vector problem. Wow! Don't you feel all warm and fuzzy inside?

Vector Components: Any vector can be drawn as the resultant of two perpendicular vectors, one along the x axis and the other along the y axis. The two vectors are perpendicular to each other and are called **component vectors**. Turning a single vector into two component vectors is called “resolving the vector into its x and y component vectors”. This is done using trigonometry. Here's an example:



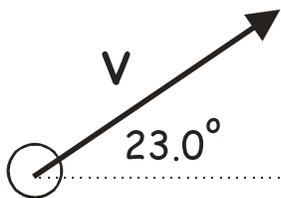
Vector A is the one we start with. It is made up of two vectors that lie in the x and y plane. We call the one along the x axis “ A_x ” and the one along the y axis is “ A_y ”. Using trig we see that:

$$A_x = A \cos \theta \quad A_y = A \sin \theta$$

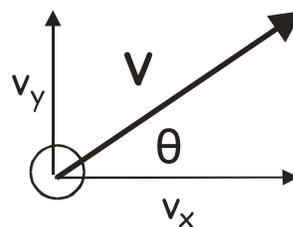
The angle θ can also be easily found:

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

- A ball is thrown with a velocity of 22.5 m/s at an angle of 23.0° to the horizon. What are its vertical and horizontal velocity components?



The components look like this:



Using trig to solve the thing we get:

$$v_x = v \cos \theta = 22.5 \frac{m}{s} \cos 23.0^\circ \quad v_x = \boxed{20.7 \frac{m}{s}}$$

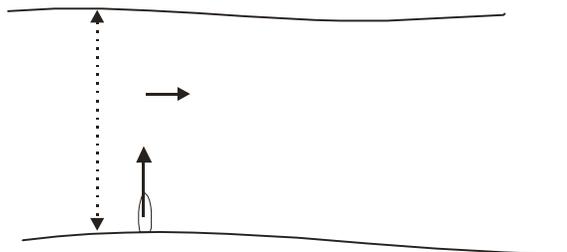
$$v_y = v \sin \theta = 22.5 \frac{m}{s} \sin 23.0^\circ \quad v_y = \boxed{8.79 \frac{m}{s}}$$

More Vector Problems:

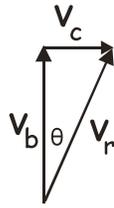
- A velocity vector has the following components: v_y is 12.4 m/s and v_x is 15.4 m/s, find the angle θ .

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{12.4 \frac{m}{s}}{15.4 \frac{m}{s}} \right) = \boxed{38.8^\circ}$$

- A boat is traveling north directly across a river that is 5.0 km across. The current is to the east at 3.0 km/h. The boat travels at 5.0 km/h. (a) What is the boat's resultant velocity? (b) How long does it take to cross the river? (c) How far down the bank does it drift before it reaches the far side?



(a) Here is a vector addition drawing, it gives one a basic idea of what the answer looks like.



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irst we find resultant velocity: $c^2 = a^2 + b^2$ $c = \sqrt{a^2 + b^2}$

$$v_r = \sqrt{\left(3.0 \frac{km}{h}\right)^2 + \left(5.0 \frac{km}{h}\right)^2} = \boxed{5.8 \frac{km}{h}}$$

(b) Find the time to cross: the velocity component that is in the direction to cross the river determines how quickly the boat crosses, so use v_b .

$$v = \frac{x}{t} \quad t = \frac{x}{v} \quad t = 500.0 \cancel{m} \left(\frac{1}{5.0 \frac{km}{h}} \right) \left(\frac{1 \cancel{km}}{1000 \cancel{m}} \right) = \boxed{0.100 h}$$

(c) Find distance in horizontal direction:

$$v = \frac{x}{t} \quad x = vt \quad x = 3.0 \frac{km}{h} (0.10 h) = \boxed{0.30 km}$$