

# AP Physics – Electric Potential Energy

Indulge the Physics Kahuna for a bit of a quick review of some vital previously covered material. The importance of the earlier concepts will be made clear as we proceed.

Work takes place when a force acts over a distance.

$$W = F r$$

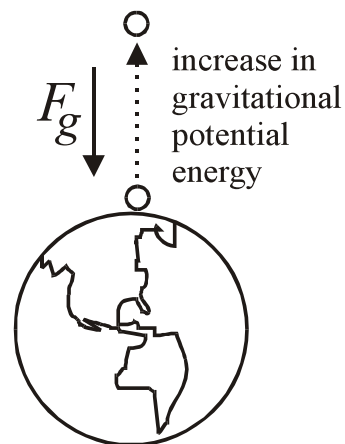
The unit for work is the joule (J). A joule is a newton · meter.

Potential energy from gravity is  $U_g = mgh$ .

The force of gravity acting on an object with mass is always directed towards the center of the earth.

Remember?

When you lift an object you do work on it. As its height is increased, its potential energy increases. The work that you do lifting it is equal to the increase in potential energy of the thing.



Imagine a skyscraper. An object that is on the 75 floor has more potential energy than does an object on the 2<sup>nd</sup> floor, true?

If we move an object from the 10<sup>th</sup> floor to the 20<sup>th</sup> floor, we've (a) done work on it and (b) increased its potential energy.

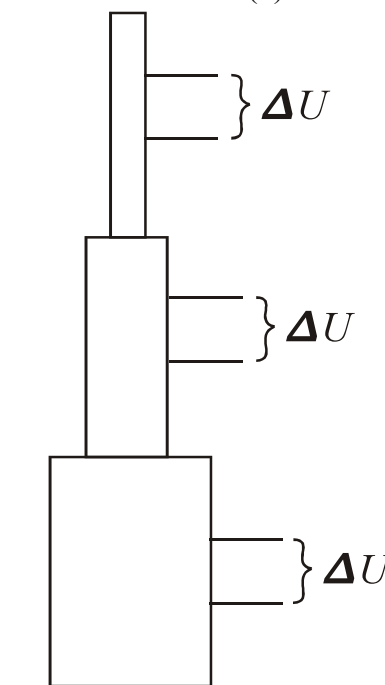
If we move an identical object from the 25<sup>th</sup> floor to the 35<sup>th</sup> floor, we've (a) done work on it and (b) increased its potential energy.

In fact we've done the same amount of work on both occasions and we've increased the potential energy of the object by the same amount, true?

We've done ten floors worth of work and increased the potential energy ten floors worth.

We could make it even more general by looking at potential energy changes per kilogram. Then we wouldn't have to worry about the objects all having the same mass. We would be dealing with a change in potential energy per kilogram.

Analyzing changes in gravitational potential energy doesn't turn out to be terribly useful in dealing with gravity, energy, and work. So why did the Physics Kahuna waste your precious time with the pointless discussion?



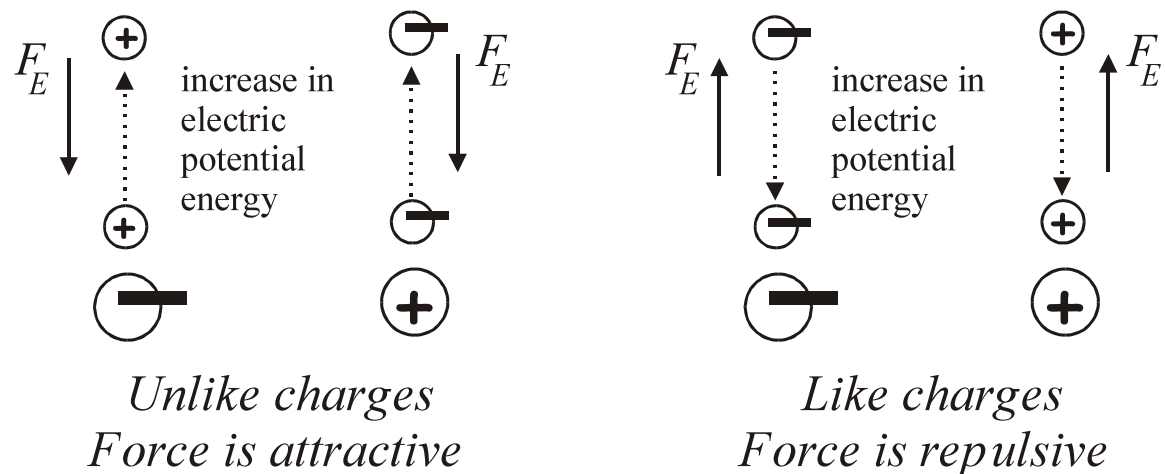
*For the same height change, potential energy change is the same*

Well, it isn't useful for gravity fields, but it is terribly useful when dealing with electric fields.

Looking at an electric field around a charged object we find that we also have work being done and potential energy changes being made when we move charged objects about.

We have a comparable situation to that of gravity when we have two opposite charges. We have a large charge  $Q$ . Placed near this charge is a small test charge  $q$ . As we increase the distance between  $q$  and  $Q$ , we have to do work to move the charge and we increase its potential energy.

If the charges are the same, then we have a different situation. It takes work to move the test charge closer to the other charge (the force is repulsive between them). As the two charges move closer together, the potential energy increases.



**Electric Potential Energy:** Let's go ahead and develop an equation for the potential energy of a charge in an electric field.

Let's look a simple situation with two charges. Now, deriving an equation for electrical potential energy sounds too hard. We haven't studied anything about electrical potential energy, have we? Oh, ye, of little faith. We don't need to study anything special, we already know all that we need to in order to figure this out.

We know that potential energy is force times distance.

$$U = Fr$$

The force acting between the electron and the proton is:

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$$

We can plug in Coulomb's law for the force in the potential energy equation.

$$U = \left( \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2} \right) r = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

So there it is. An equation for electrical potential energy:

$$U_E = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

- An electron in a hydrogen atom is at its lowest energy state. It is at a distance of  $5.29 \times 10^{-11}$  m from the proton in the nucleus. What is its potential energy relative to the proton?

We can look up the charges for the particle, although by now, you probably know what they are by heart. Anyway, here are the charges:

$$q_1 = -1.60 \times 10^{-19} \text{ C} \quad q_2 = 1.60 \times 10^{-19} \text{ C}$$

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

$$U = 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left(1.60 \times 10^{-19} \text{ C}\right) \left(1.60 \times 10^{-19} \text{ C}\right) \left(\frac{1}{5.29 \times 10^{-11} \text{ m}}\right)$$

$$U = \boxed{4.35 \times 10^{-18} \text{ J}}$$

**Work in the Electric Field:** Earlier we mentioned that work was done when a particle was moved from one point to another in an electric field. This is accompanied by an increase in the charge's potential energy (the charge that gets moved).

If the two charges are unlike:

Pull the charges apart – increase potential energy (just like with gravity).

Push the charges together - potential energy decreases.

If the two charges are the same:

Pull the charges apart - decrease the potential energy.

Push together - potential energy increases.

The important thing here is that the work done in moving the charge is equal to the potential energy difference between the two points

The critical thing is the change in displacement – the difference between the starting point and the final position of the charge. We don't care about what the actual distance between the charges is.

We know that work is:  $W = F r$

The force exerted on a test charge in an electric field is:

$$E = \frac{F}{q} \qquad F = qE$$

We now put these two equations together and we get:

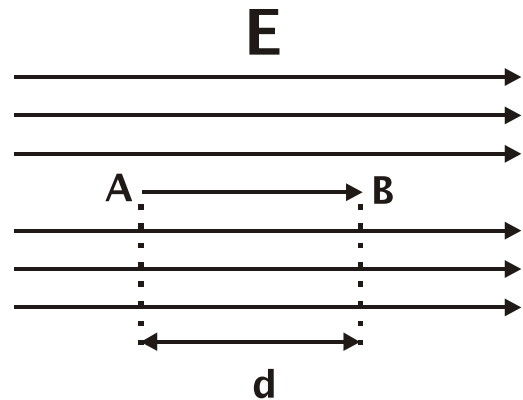
$$W = Fd = (qE)d = qEd$$

This gives us an equation for the work done in an electric field in moving a charge from one position to another:

$$W = qEd$$

$W$  is the work done in Joules,  $q$  is the charge of the test charge (not the charge making the field),  $E$  is the electric field strength, and  $d$  is the displacement of the charge.

In the drawing to the right, we have a uniform electric field of strength  $E$ . A charge is moved from point  $A$  to point  $B$ , which is a distance  $d$ . The work needed to do this is given by the equation we just developed.



The magnitude of the work done in moving the charge is equal to the change in potential energy. Therefore we can write:

$$\Delta U = -W$$

The negative sign simply means that the potential energy will increase if the charge is negative or decrease if the charge is positive.

We recall the equation for work:

$$W = qEd$$

We plug this into the equation for the change in potential energy.

$$\Delta U = -qEd$$

This is another useful equation.

**Potential Difference:** We can make things even more general. Instead of looking at the total potential energy, we can look at the potential energy per Coulomb of charge.

This, the potential energy per charge, is called *potential difference*.

**Potential Difference = Change in potential energy per Charge**

The symbol for potential difference is  $\Delta V$  or  $V$ .

$$V \equiv \text{Potential Difference}$$

$$V = \frac{U}{q}$$

If we look at the equation for potential energy and divide both sides by the charge  $q$ , then we get the potential difference.

$$U = -qEd \quad \text{We divide both sides by } q \text{ (which is mathematically legal).}$$

$$\frac{U}{q} = \frac{-\cancel{q}Ed}{\cancel{q}} \quad V = -Ed$$

This equation is commonly rearranged to this form:

$$E = -\frac{V}{d}$$

This is the equation that you will have available for the AP Physics Test. Notice also that the electric field can be expressed in units of V/m as well as N/C.

Here is the formal definition for potential difference between points  $a$  and  $b$ :

**Potential difference is the change in potential energy as a charge  $q$  is moved from  $a$  to  $b$  divided by the charge  $q$ .**

If  $V = -Ed$ , then

$$U = -qEd \quad V = -Ed \quad \text{so} \quad U = qV$$

On the AP Physics Exam, the equations for potential energy are combined together into one, looks like this:

$$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

And of course, the work is equal to the change in potential energy, so

$$W = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

The unit for potential difference is the **volt** (V). The volt is a joule/Coulomb.

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

A potential difference of one volt means that it takes one joule of energy to move a one Coulomb charge in the field. A potential difference of 1000 V means that it would take 1000 J to move 1 C of charge in the field.

Potential difference is also called **electric potential** or **voltage**.

Potential difference is one of the most important concepts in physics when you are dealing with electricity, so you want to take special care to master the thing. It is essential that you have a solid understanding about potential difference and how it works.

Charge moves because of the potential difference,  $\Delta V$ . The bigger the potential difference between two points, the more the charge will want to move.

Electric potential is a scalar quantity.

Here are the equations that you will have to work with on the AP Physics Test:

$$E = \frac{F}{q} \qquad U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r} \qquad E_{Avg} = -\frac{V}{d}$$

We'll now make use of them to solve the odd problem or two.

- The potential difference between two points is 12.0 V. What amount of work is needed to move a 2.00  $\mu\text{C}$  charge within the field?

$$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r} \qquad U_E = qV = W$$

$$W = qV = 2.00 \times 10^{-6} \text{ C} (12.0 \text{ V}) = 24.0 \times 10^{-6} \text{ J} = \boxed{2.40 \times 10^{-5} \text{ J}}$$

- Two charged plates are 5.00 cm apart. The electric field between them is 775 N/C. What is (a) the potential difference between the plates and (b) what work is done moving an electron from one plate to another?

$$(a) E = -\frac{V}{d} \quad V = -Ed = -775 \frac{N}{C} (0.050 \text{ m}) = -38.8 \frac{J}{C} = \boxed{-38.8 \text{ V}}$$

(b) We know the electron's charge, so this is fairly simple:

$$W = qV = 1.60 \times 10^{-19} \text{ C} (38.8 \text{ V}) = 62.1 \times 10^{-19} \text{ J} = \boxed{6.21 \times 10^{-18} \text{ J}}$$

- The voltage between two big charged up parallel plates is -120.0 V. The plates are 2.50 cm apart. What is the electric field between them?

$$E = -\frac{V}{d} = -\left(-120.0 \frac{J}{C}\right) \frac{1}{0.0250 \text{ m}} = 4800 \frac{N}{C} = \boxed{4.80 \times 10^3 \frac{N}{C}}$$

Notice that the Physics Kahuna has abandoned showing how the units all work out. This is because in the electric world with all of its wacky units, it is just too much trouble.

Here's what you do. You put everything into standard units: meters, Newtons, joules, Coulombs, Volts, &c. Then you just put down the proper unit that the answer is supposed to have.

- A proton is released from rest in a uniform electric field,  $E = 8.0 \times 10^4 \text{ V/m}$ . It is directed along the  $x$  axis. It's displacement is 0.50 m in the direction of the field. (a) What is the change in potential difference? (b) What is the change in electrical potential energy? (c) What is its velocity after it traveled the 0.50 m?

(a) We can find the potential difference since we know the electric field strength and the displacement.

$$E = -\frac{V}{d} \quad V = -Ed = -\left(8.0 \times 10^4 \frac{V}{\cancel{\text{m}}}\right) (0.50 \cancel{\text{m}}) = \boxed{-4.0 \times 10^4 \text{ V}}$$

(b) The potential energy is easy to find. We know the charge of a proton and we know the potential difference it is exposed to.

$$U_E = qV = \left(1.60 \times 10^{-19} \cancel{\text{C}}\right) \left(-4.0 \times 10^4 \frac{J}{\cancel{\text{C}}}\right) = \boxed{-6.40 \times 10^{-15} \text{ J}}$$

(c) The proton's potential energy will be converted into kinetic energy as it moves through the 0.50 m. So we can set the kinetic energy equal to the potential energy and solve for the particle's velocity.

$$U = \frac{1}{2}mv^2 \quad v = \sqrt{\frac{2U}{m}} \quad \text{We can look up the mass of a proton.}$$

$$v = \sqrt{2 \left( 6.40 \times 10^{-15} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \right) \left( \frac{1}{1.67 \times 10^{-27} \text{kg}} \right)}$$

$$v = \sqrt{7.66 \times 10^{12} \frac{\text{m}^2}{\text{s}^2}} = \boxed{2.8 \times 10^6 \frac{\text{m}}{\text{s}}}$$

**Potential Difference and Conductors:** In a conductor electrons, for all practical purposes, are free to move about every which way but loose. This means that there is no potential difference within a conductor from one point to another. If one end of a wire has a potential difference of 12 V, the other end will also have a potential of 12 V.

We physics types say that all points in a conductor are at an equipotential. When conductors are joined together, the charge ends up being evenly distributing throughout the newly formed conductive system. The new conductor system will also be at an equipotential.