AP Physics - Coulomb's Law

We've learned that electrons have a minus one charge and protons have a positive one charge. This plus and minus one business doesn't work very well when we go in and try to do the old major figuring stuff out deal– it's kind of arbitrary thing, see. So we need a really useful unit of charge.

Well, we got one. It is your basic standard unit of charge; a thing called a ***Coulomb*** (C).

The symbol for charge is Q however q is used as well.

One Coulomb is equal to the charge of 6.25×10^{18} electrons or protons.

The charge of a single electron is -1.60×10^{-19} C. The charge of a proton is $+1.60 \times 10^{-19}$ C.

The Coulomb is a large amount of charge, so it is very common to use micro Coulombs and milli Colulombs.

$1 \text{ mC} = 10^{-3} \text{ C}$

$1 \text{ µC} = 10^{-6} \text{ C}$

- What is the charge of 1.35×10^{17} electrons?

This is a simple dimensional analysis type o'problemo.

$$1.35\,x\,10^{17}\cancel{e}\left(\frac{1.60\,x\,10^{-19}C}{1\cancel{e}}\right) = \boxed{2.06\,x\,10^{-2}C}$$

or

$$2.06\,x\,10^{-2}\cancel{C}\left(\frac{1\,\mu C}{1\,x\,10^{-3}\cancel{C}}\right) = 2.06\,x\,10^{1}\,\mu C = \boxed{20.6\,\mu C}$$

Coulomb's Law: Charged objects exert forces on one another. This is very similar to what happens with gravity between two objects that have mass. Recall that Newton's universal law of gravity can be used to calculate the force between two objects that have mass. It turns out that there is a similar law that can be used to calculate the force between two objects that have charge.

This law is called ***Coulomb's law***. Here it is:

$$F = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r^2}$$

F is the force exerted between the two charges. q_1 and q_2 are the two charges. (Note, we will actually use the absolute value of the charges - we simply don't care about whether they are positive or negative.) ***r*** is the distance between the two charges and $\frac{1}{4\pi \in_0}$ is called Coulomb's Constant. It is similar to the universal gravitational constant.

The value for Coulomb's constant is:

$$\frac{1}{4\pi \in_0} = 8.99 \, x \, 10^9 \frac{Nm^2}{C^2}$$

Coulomb's law in most physics books is usually written in a slightly different form:

$$F = \frac{k_e |q_1||q_2|}{r^2} \qquad or \qquad F = \frac{k_e q_1 q_2}{r^2}$$

where

$$k_e = \frac{1}{4\pi \in_0} = 8.99 \, x \, 10^9 \frac{Nm^2}{C^2}$$

But we won't use that form, because the wonderful AP Physics folks use the first one that the Physics Kahuna gave you.

The force between two charged objects can be either attractive or repulsive, depending on whether the charges are like or unlike.

We will also assume that the charges are concentrated into a small area – ***point charges***.

Example Problems:

- Two point charges are 5.0 m apart. If the charges are 0.020 C and 0.030 C, what is the force between them and is it attractive or repulsive?

$$F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2} = 8.99 \, x \, 10^9 \frac{N\cancel{m}^2}{\cancel{C}^2} \left(\frac{(0.020 \, \cancel{C})(0.030 \, \cancel{C})}{(5.0 \, \cancel{m})^2} \right)$$

$$F = 0.000216 \, x \, 10^9 \, N = \boxed{2.2 \, x \, 10^5 \, N}$$

The force is repulsive - both charges are positive.

- A force of 1.6×10^{-3} N exists between 2 charges; 1.3 μC and 3.5μC. How far apart are they?

$$F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} \qquad r = \sqrt{\frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{F}}$$

$$r = \sqrt{\frac{8.99 \, x \, 10^9 \frac{\cancel{N} m^2}{\cancel{C}^2} \left(1.3 \, x \, 10^{-6} \cancel{C}\right)\left(3.5 \, x \, 10^{-6} 6 \cancel{C}\right)}{1.6 \, x \, 10^{-3} \cancel{N}}}$$

$$r = \sqrt{25.57 \, x \, 10^0 m^2} \quad = \quad \boxed{5.1 \, m}$$

Electric Force and Gravity: Both gravity and the electric force are fundamental forces.

The equations for the gravity and the electromagnetic force have the same form; they are both inverse square relationships.

$$F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} \qquad \text{and}$$

$$F = \frac{G \, m_1 m_2}{r^2}$$

Where the $\frac{1}{4\pi \epsilon_0}$ term is the constant for Coulomb's law and ***G*** is the constant for the law of gravity.

There are four really significant differences in the two forces:

- Gravity is always attractive. The electromagnetic force can be either attractive or repulsive.
- Gravity is much weaker.
- Gravity has a much greater range within which it is a significant force.
- The electric force can be shielded, blocked, or cancelled. You cannot do any of these things with the gravity force.

The force of gravity is around 10^{40} times smaller than the electromagnetic force. This can be seen in a comparison of the two proportionality constants.

$$\frac{1}{4\pi \in_0} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 \text{ for the electric force}$$

$$\boldsymbol{G} = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \text{ for the gravitational force}$$

The two constants differ by a factor of 10^{20}!

Gravity extends out to great distances. The sun is 92 million miles away from the earth, yet the force of gravity is large enough to cause the earth to be locked in an orbit around the sun. Now for electricity, the force between two charges drops off very quickly with distance. This is because the magnitude of the two charges is very small – at the most, maybe a few Coulombs. But with gravity, we are dealing with enormous masses - the mass of the sun is 1.99×10^{30} kg! Because of these large masses, even though the gravity constant is very small, the force of gravity between really large masses ends up being a really big force that reaches out over distances of billions and even trillions of kilometers.

Let us compare forces in a hydrogen atom. The hydrogen atom is made up of a proton and an electron. The two particles attract each other because they both have mass and they also have opposite charges.

The magnitude of the electron/proton charge is 1.60×10^{-19} C. The distance between them in a hydrogen atom is around 5.3×10^{-11} m. For the mass of an electron we'll use 9.1×10^{-31} kg. For the mass of a proton we'll use 1.7×10^{-27} kg.

We can now calculate the force of gravity between the two particles:

$$F = \frac{Gm_1m_2}{r^2} \quad = 6.67\,x\,10^{-11}\,\frac{N\cancel{m}^2}{\cancel{kg}^2}\left(\frac{\left(1.7\,x\,10^{-27}\,\cancel{kg}\right)\left(9.1\,x\,10^{-31}\,\cancel{kg}\right)}{\left(5.3\,x\,10^{-11}\,\cancel{m}\right)^2}\right)$$

$$F = \boxed{3.7\,x\,10^{-47}\ N}$$ (Pretty small, ain't it?)

Next, we solve for the electromagnetic force.

$$F = \frac{1}{4\pi \in_0}\frac{q_1q_2}{r^2} \quad = 8.99\,x\,10^9\,\frac{N\cancel{m}^2}{\cancel{C}^2}\left(\frac{\left(1.60\,x\,10^{-19}\,\cancel{C}\right)\left(-1.60\,x\,10^{-19}\,\cancel{C}\right)}{\left(5.3\,x\,10^{-11}\,\cancel{m}\right)^2}\right)$$

$$F = 0.82\,x\,10^{-7}\,N \quad = \boxed{8.2\,x\,10^{-8}\,N}$$

Looking at the two forces, we see that gravity is much weaker. The electromagnetic force is 2.2×10^{39} times bigger!

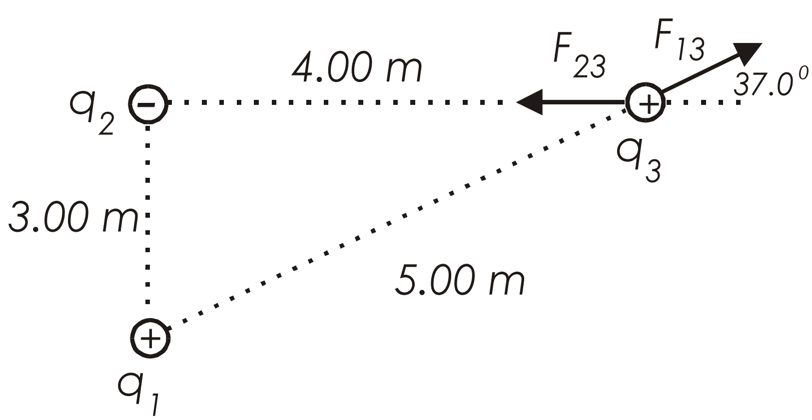
Gravity vs Electricmagnetic Force: Gravity fields and electric fields are both similar and different at the same time. Here is a handy little table to organize things:

Gravity Force	Electricmagnetic Force
Attracts	attracts and repels
inverse square law	inverse square law
surround objects	surround objects
cannot be shielded	can be shielded
incredibly weaker	enormously stronger

Superposition Principle: When we have more than two charges in proximity, the forces between them get more complicated. But, please to relax, even though things seem complicated, they actually ain't and it is fairly simple to work things out. The forces, being vectors, just have to be added up. We call this the ***superposition principle***.

> ***Superposition Principle ≡ The resultant force on a charge is the vector sum of the forces exerted on it by other charges.***

Let's look at a system of three charges. The charges are arranged as shown in the drawing. q_1 is 3.00 m from q_2. q_2 is 4.00 m from the q_3. (We immediately spot this as one of those "345" triangle deals, so we know that q_1 is 5.00 m from q_3). What is the net force acting on q_3?



q_3 is attracted to q_2 (they have opposite charges) and repulsed by q_1 (they have the same charge). The two force vectors have been drawn and labeled, F_{23} and F_{13}.

$$q_1 = 6.00 \, x \, 10^{-9} C \quad q_2 = 2.00 \, x \, 10^{-9} C \quad q_3 = 5.00 \, x \, 10^{-9} C$$

The net force on q_3 is $F_{23} + F_{13}$.

The first step is solving the problem is to find the magnitude of the 2 forces:

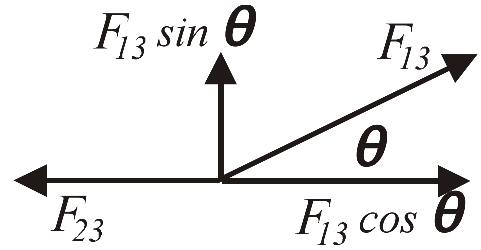
$$F_{13} = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r^2} \quad = 8.99\,x\,10^9 \frac{N\cancel{m}^2}{\cancel{C}^2}\left(\frac{\left(5.00\,x\,10^{-9}\cancel{C}\right)\left(6.00\,x\,10^{-9}\cancel{C}\right)}{\left(5.00\cancel{m}\right)^2}\right)$$

$$F_{13} = 10.8\,x\,10^{-9}\,N \quad = \quad \boxed{1.08\,x\,10^{-8}\,N}$$

$$F_{23} = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r^2} \quad = 8.99\,x\,10^9 \frac{N\cancel{m}^2}{\cancel{C}^2}\left(\frac{\left(2.00\,x\,10^{-9}\cancel{C}\right)\left(5.00\,x\,10^{-9}\cancel{C}\right)}{\left(4.00\cancel{m}\right)^2}\right)$$

$$F_{23} = \quad \boxed{5.62\,x\,10^{-9}\,N}$$

The next step is to break the two vectors down into their horizontal and vertical components and add the two vectors in the ***x*** and ***y*** directions. This gives us the components of the resultant vector, $\boldsymbol{F_X}$ and $\boldsymbol{F_Y}$:



$$F_x = F_{13}\cos\theta - F_{23}$$

$$F_x = \left(1.08\,x\,10^{-8}\,N\right)\cos 37.0^\circ - 5.62\,x\,10^{-9}\,N$$

$$F_x = 8.63\,x\,10^{-9}\,N - 5.62\,x\,10^{-9}\,N \quad = \quad \boxed{3.01\,x\,10^{-9}\,N}$$

$$F_y = F_{13}\sin\theta$$

$$F_y = \left(1.08\,x\,10^{-8}\,N\right)\sin 37.0^o \quad = \quad \boxed{6.50\,x\,10^{-9}\,N}$$

Now we can find the resultant force:

$$F = \sqrt{F_y^{\,2} + F_x^{\,2}} \quad = \sqrt{\left(6.50\,x\,10^{-9}\,N\right)^2 + \left(3.01\,x\,10^{-9}\,N\right)^2} \quad = \sqrt{51.31\,x\,10^{-18}\,N^2}$$

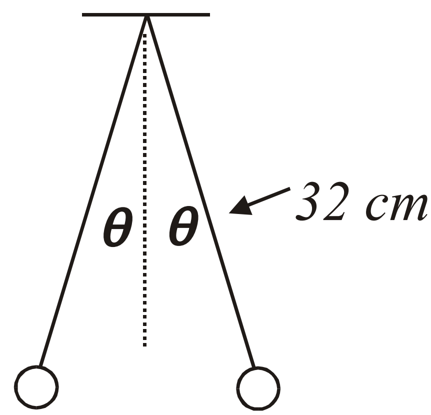
$$F = \quad \boxed{7.16\,x\,10^{-9}\,N}$$

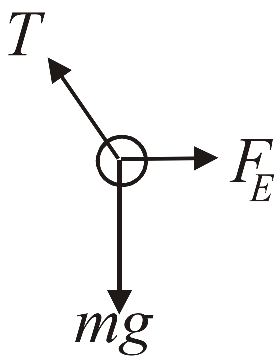
Now we can find the direction or the resultant force:

$$\theta = \tan^{-1}\left(\frac{F_Y}{F_X}\right) \qquad \theta = \tan^{-1}\left(\frac{6.50\,x\,10^{-9}\cancel{N}}{3.01\,x\,10^{-9}\cancel{N}}\right) \qquad \theta = 65.2^\circ \text{ with the x axis}$$

Isn't it great to solve these problems? Let's do another!

- Two 0.25 g metal spheres have identical positive charges. They hang down from light strings that are 32 cm long commonly attached to the ceiling. If the angle the strings form with the vertical is 7.0°, what is the magnitude of the charges?





First let's draw a FBD for the forces on one of the spheres:

There are three forces: the tension ***T*** in the string holding the sphere up, the electric force $\boldsymbol{F_E}$, and the weight of the sphere ***mg***.

Because the system is static, the sum of the forces must be zero. The forces in the ***x*** direction are:

$$F_E - T\sin\theta = 0$$

The forces in the ***y*** direction are:

$$T\cos\theta - mg = 0$$

Now things become simpler, we can use this last equation to find the tension. Armed with the tension we can find the electric force. Using the electric force we can find the charge on the sphere. Let's go ahead and find the tension.

$$T\cos\theta - mg = 0 \quad T = \frac{mg}{\cos\theta} \quad = (0.025\ kg)\left(9.8\frac{m}{s^2}\right)\left(\frac{1}{\cos 7.0^{\circ}}\right) \quad = \quad 0.247\ N$$

Now we find $\boldsymbol{F_E}$:

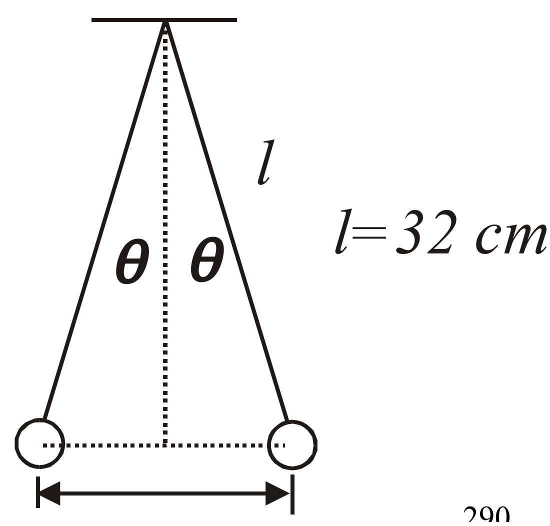
$$F_E - T\sin\theta = 0 \quad F_E = T\sin\theta \quad = (0.247\ N)\sin 7.0^{\circ} \quad = \quad 0.0301\ N$$

Coulomb's law is next:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

The 2 charges have the same value; we need to find the distance between the two spheres ***r***.

We can see that half of r is $l\sin\theta$. So r is $2l\sin\theta$.



$$F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$$

But the two charges are the same, so:

$$F = \left(\frac{1}{4\pi \in_0}\right)\frac{qq}{r^2} = \left(\frac{1}{4\pi \in_0}\right)\frac{q^2}{r^2}$$

$$q^2 = Fr^2\left(\frac{1}{\frac{1}{4\pi \in_0}}\right)$$

$$q = r\sqrt{F\left(\frac{1}{\frac{1}{4\pi \in_0}}\right)} = 2l\sin\theta\sqrt{F\left(\frac{1}{\frac{1}{4\pi \in_0}}\right)}$$

$$q = 2(0.32\ \cancel{m})\sin 7.0^{o}\sqrt{(0.0301\ \cancel{N})\left(\frac{1}{8.99\,x\,10^{9}\,\frac{\cancel{Nm}^2}{C^2}}\right)}$$

$$q = 0.0780\sqrt{0.003348\,x\,10^{-9}\ C^2} = 0.0780\sqrt{0.03348\,x\,10^{-10}\ C^2} = 0.0143\,x\,10^{-5}\ C$$

$$q = \boxed{1.43\,x\,10^{-7}\ C}$$