

Sample exam II Solutions

1) F always points the same direction as a , hence as Δv b/c $\vec{a} = \frac{\Delta \vec{v}}{t}$

E wrong b/c F and Δv must be same way

2)

$$P = \frac{W}{t} \quad W = \Delta K = \frac{1}{2} m v^2$$

$$P = \frac{\frac{1}{2} m v^2}{t} \quad v_2 = \sqrt{2} v_1$$

or

$$\frac{\cancel{P}_1 = \frac{\frac{1}{2} \cancel{m} v_1^2}{\cancel{t}}}{\cancel{P}_1 = \frac{\frac{1}{2} \cancel{m} v_2^2}{2\cancel{t}}} \Rightarrow 1 = \frac{2 v_1^2}{v_2^2}$$

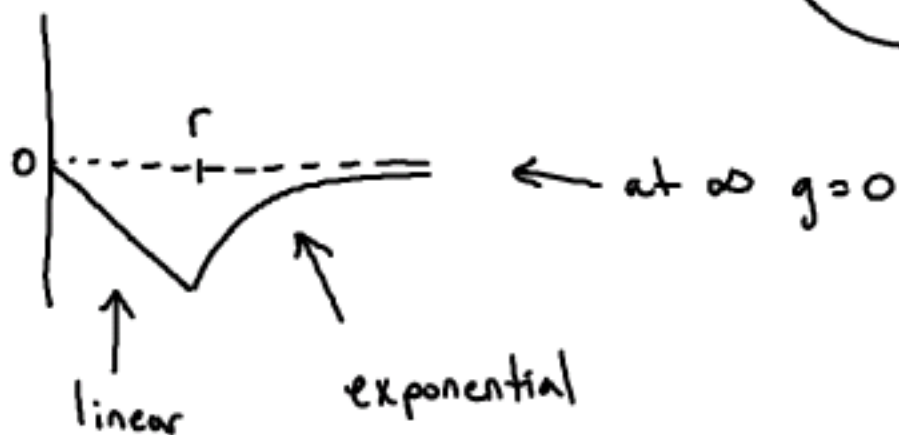
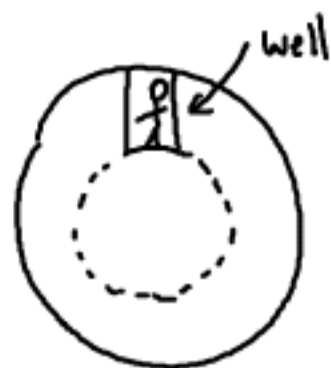
$$v_2^2 = 2 v_1^2$$

$$v_2 = \sqrt{2} v_1$$

3) g vs. r

assume + is away from earth

$$g = G \frac{m}{r^2}$$



4) SHM

$$x = A \cos \omega t$$

$$a = -A \omega^2 \cos \omega t$$

$$a = -\omega^2 \underline{A \cos \omega t}$$

$$a = -\omega^2 x$$

$$\frac{d^2 x}{dt^2} = -\omega^2 x$$

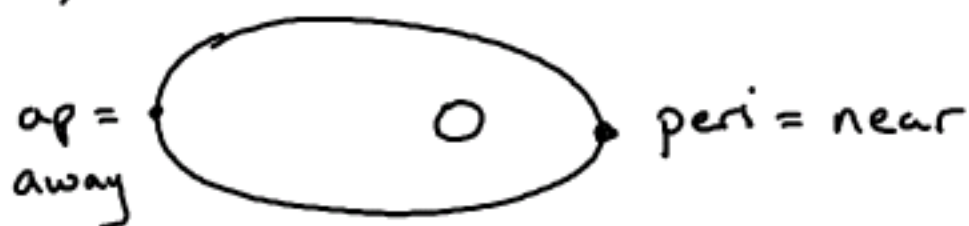
match

negative

constant

Key parts

5)



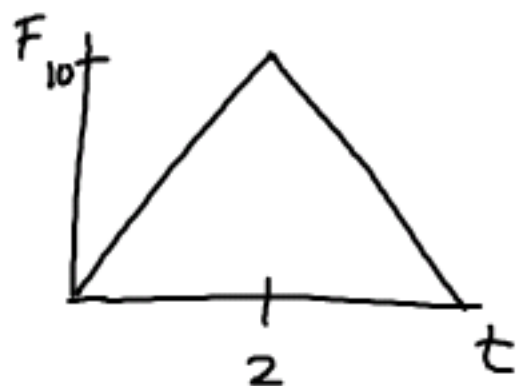
$$L_i = L_f$$

$$\cancel{m_1 v_1 r_{\perp 1}} = \cancel{m_1 v_2 r_{\perp 2}}$$

$$V_p \cdot 4.6 \times 10^{10} = V_A \cdot 7.0 \times 10^{10}$$

$$V_p = \frac{V_a \cdot 7.0 \times 10^{10}}{4.6 \times 10^{10}}$$

6)



F vs. t

$$\int F dt = J = \Delta mv$$

$$\text{area} = 20 \text{ units}$$

$$v_0 = 5$$

$$m = 1$$

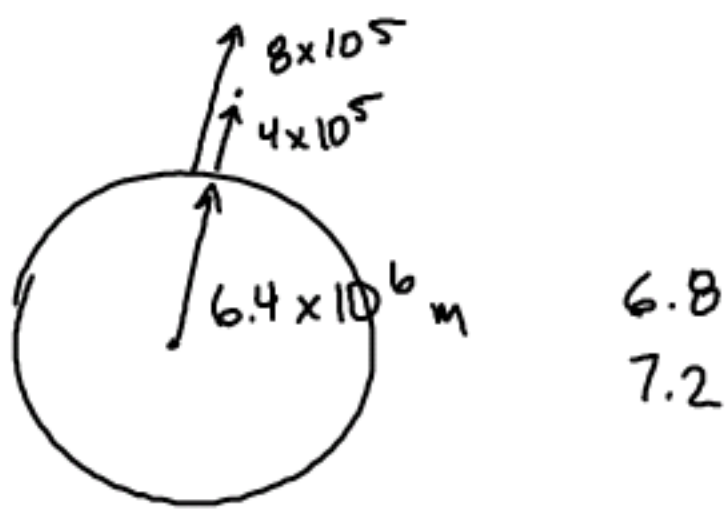
$$20 = \Delta mv$$

$$20 = 1 \Delta v$$

$$v_f = 25 \frac{m}{s}$$

$$F \cdot t = \Delta mv$$

7)



$$V = \sqrt{\frac{GM}{r}}$$

$$\frac{V_2}{V_0} = \frac{\sqrt{\frac{GM_e}{7.2 \times 10^6}}}{\sqrt{\frac{GM_e}{6.8 \times 10^6}}}$$

$$V_2 = V_0 \sqrt{\frac{6.8}{7.2}}$$

8)

$$V = 10t - 2t^2$$

$V_t = ?$ $V_t @ a = 0$ find a

$$\frac{dv}{dt} = a = \frac{d}{dt}(10t - 2t^2)$$

$$a = 10 - 4t - \text{set } a = 0$$

find t

$$0 = 10 - 4t$$

$$\frac{10}{4} = t = 2.5s - \text{plug } t \text{ into}$$

V eq.

$$V = 10t - 2t^2$$

$$V_t = 10(2.5) - 2(2.5)^2 = 12.5 \frac{m}{s}$$

9)

$$a = 3x$$

$$V = ? \text{ when } x = 5$$

$$a = \frac{dv}{dt} = 3x$$

multiply by 1

$$\frac{dv}{dt} \cdot \left(\frac{dt}{dx} \cdot v \right) = 3x$$

$$\frac{dv}{dx} v = 3x \rightarrow \int v dv = \int 3x dx$$

$$\frac{v^2}{2} + \cancel{c} = \frac{3x^2}{2}$$

$$v^2 = 3x^2$$

$$v = \sqrt{3} x$$

(or \rightarrow)

9) alternate solution

$$a = 3x$$

$$W = \int F dx = \Delta K$$

$$\int ma dx = \Delta K$$

$$\int m(3x) dx = \frac{1}{2}mv^2$$

$$\cancel{m} \frac{3x^2}{2} = \frac{1}{2} \cancel{m} v^2$$

$$\sqrt{3}(x) = v$$

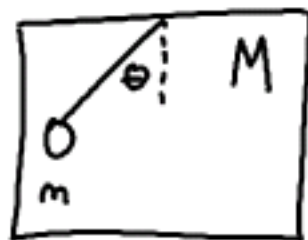
$$10) \quad \Theta = 3t + 2t^2 \quad v = r\omega$$

$$\omega = \frac{d\Theta}{dt} = \frac{d}{dt}(3t + 2t^2) = 3 + 4t$$

$$\omega_5 = 3 + 4(5) = 23$$

$$v = (0.3)(23) = 6.9 \frac{m}{s}$$

11)



$$\sum F_y = m a_y = 0$$

$$T \cos \theta = mg$$

$$T = \frac{mg}{\cos \theta}$$

$$\sum F_x = m a_x$$

$$T \sin \theta = ma$$

$$\frac{mg}{\cos \theta} \sin \theta = ma$$

$$g \tan \theta = a$$

12)

inelastic collision $L_i = L_f$ b/c of rotate

$$L_i = L_f$$

$$mvr_{\perp} = I\omega$$

$$mv l = l^2 \left(\frac{M}{3} + m \right) \omega$$

$$\frac{mv l}{l^2 \left(\frac{M}{3} + m \right)} = \omega$$

$$I_{\text{TOT}} = I_{\text{rod}} + I_{\text{block}}$$

$$= \frac{1}{3} M l^2 + m l^2$$

$$= l^2 \left(\frac{M}{3} + m \right)$$

$$\frac{3}{3} \frac{mv}{l \left(\frac{M}{3} + m \right)}$$

$$\frac{3mv}{l(M+3m)} = \omega$$

13)

$$I_{||} = I_{cm} + md^2$$

$$= MR^2 + MR^2$$

$$= 2MR^2$$

$$14) \quad J = \int F dt = \Delta mv$$

$$-30 = \Delta mv = 5(v_f - 2)$$

$$\frac{-30}{5} = v_f - 2$$

$$-6 + 2 = v_f$$

$$-4 = v_f$$

↑
opposite original direction

15) $x = 20t + 30t^2 \leftarrow x = v_0 t + \frac{1}{2} a t^2$

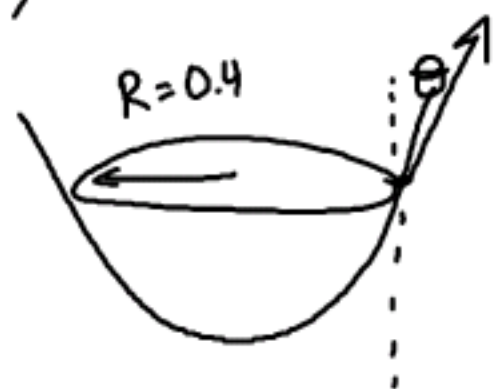
$y = 25 - 40t \leftarrow y = y_0 + v_0 t$

a in x direction only

$F = ma \quad a = 60 \quad \text{b/c } \frac{1}{2} a = 30 \text{ above}$

$F = 1(60) = 60 \text{ N}$

17)

horizontal circle

$$F_c = \sum F_r = m \frac{v^2}{r}$$



$$F_c = F_N \cos \theta$$

$$\sum F_y = 0$$

$$F_N \sin \theta = mg$$

$$F_c = \frac{mg}{\sin \theta} \cdot \cos \theta$$

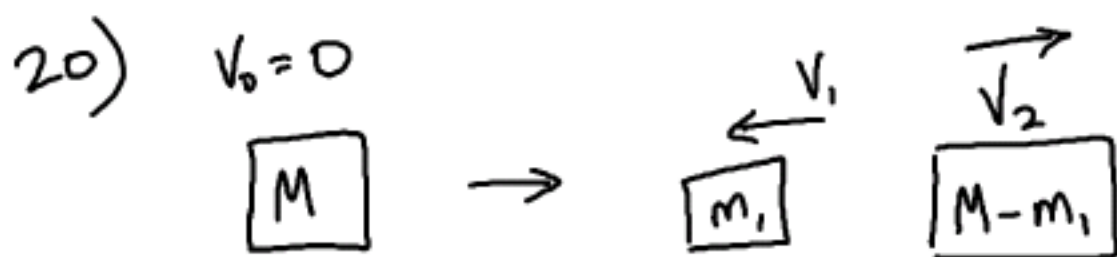
$$F_N = \frac{mg}{\sin \theta}$$

$$m \frac{v^2}{r} = mg \cot \theta$$

want ω

$$\therefore r \omega^2 r = r g \cot \theta$$

$$\omega = \sqrt{\frac{g \cot \theta}{r}}$$



$$K_1 = \frac{1}{2} m_1 v_1^2$$

$$P_i = P_f$$

$$0 = m_1 v_1 + (M - m_1) v_2$$

$$v_2 = \frac{-m_1 v_1}{(M - m_1)} \rightarrow K_2 = \frac{1}{2} (M - m_1) v_2^2$$

$$K_2 = \frac{1}{2} (M - m_1) \left(\frac{-m_1 v_1}{M - m_1} \right)^2$$

$$K_1 = \frac{1}{2} m_1 v_1^2$$

given



Sub in

for $\frac{1}{2} m_1 v_1^2$ in

eq. for K_2

$$K_2 = \frac{1}{2} \cancel{(M - m_1)} \left(\frac{m_1^2 v_1^2}{\cancel{(M - m_1)}} \right)$$

$$K_2 = \frac{K_1 m_1}{(M - m_1)}$$

22)

$$\tau_{\text{net}} = 0 \quad \therefore \alpha = 0$$

$F_{\text{net}} = ? \rightarrow$ not necessarily
could be thru pivot

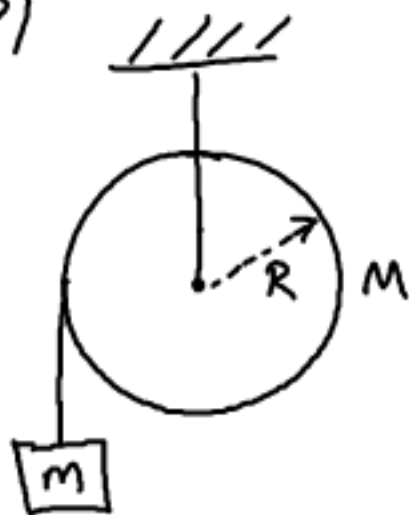
ω constant? \rightarrow not necessarily
could be non-rigid
object and I may
change as it
rotates \therefore

$$I_1 \omega_1 = I_2 \omega_2$$

Linear mom constant? - not

necessarily, one
object can have
an impulse applied
to it.

23)

block

$$\Sigma F = ma$$

$$mg - T = ma$$

$$T = mg - ma$$

pulley

$$\Sigma \tau = I\alpha$$

$$rT = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$

$$T = \frac{1}{2}Ma$$

$$\frac{1}{2}ma = mg - ma$$

$$\frac{3}{2}ma = mg$$

$$a = \frac{2}{3}g$$

24)



note: the center of mass never moves unless an outside force acts.

$$P_i = P_f$$

$$0 = m_2 v_2 + m_1 v_1$$

$$- m_1 v_1 = m_2 v_2$$

$$\frac{v_1}{v_2} = -\frac{m_2}{m_1}$$

25)

$$T_S = 2\pi \sqrt{\frac{m}{k}}$$

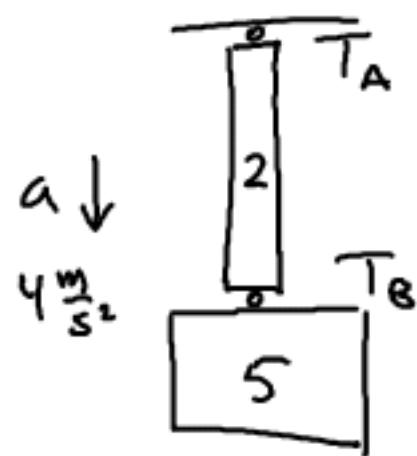
Use m_{eff}

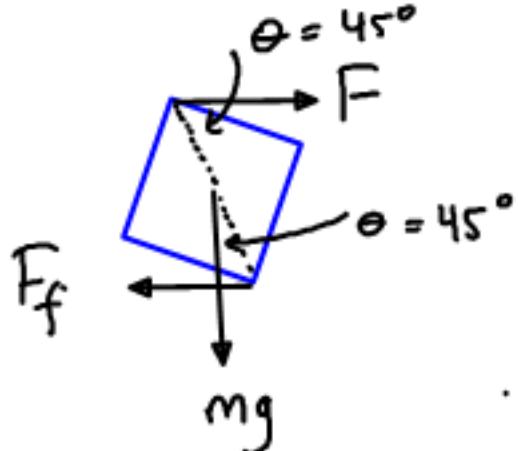
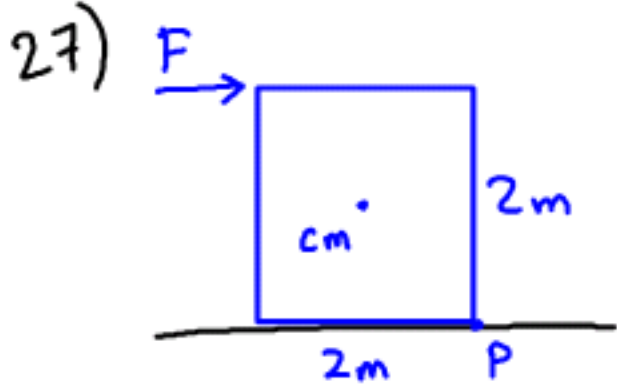
$$m_{\text{eff}} = \frac{m_1 \cdot m_2}{m_1 + m_2}$$

$$T_S = 2\pi \sqrt{\frac{\frac{m_1 \cdot m_2}{m_1 + m_2}}{k}}$$

$$= 2\pi \sqrt{\frac{m_1 \cdot m_2}{k(m_1 + m_2)}}$$

26)





use θ from
first starting

$$\sum \tau = I \alpha \quad \text{let } \alpha = 0$$

$$r mg \sin \theta - 2r F \sin \theta = 0$$

~~$$r mg \sin \theta = 2r F \sin \theta$$~~

$$\frac{mg}{2} = F$$

28)

$$r_m = \frac{1}{2} r_e \quad - \quad G \frac{m_1 m_2}{r} + \frac{1}{2} m v^2 = 0$$

$$m_m = \frac{1}{10} m_e \quad G \frac{m_1 m_2}{r} = \frac{1}{2} m v^2$$

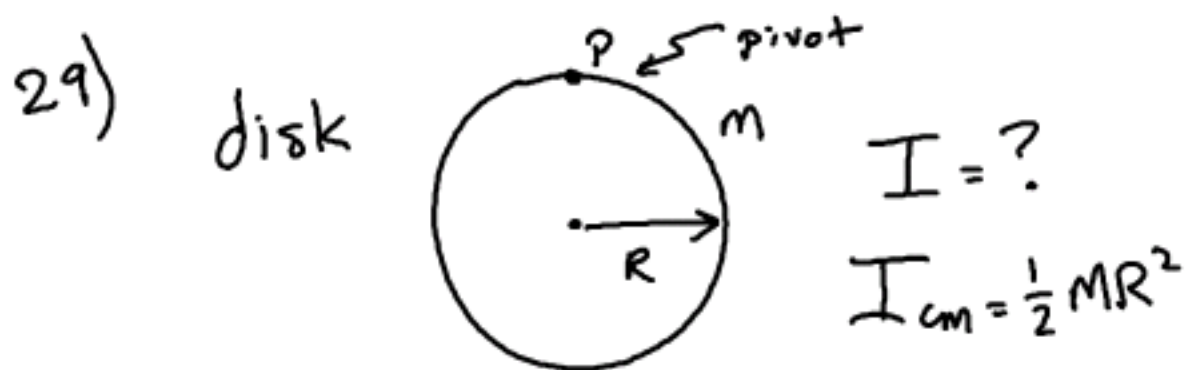
$$v_{em} = ?$$

$$v_{esc} = \sqrt{\frac{2 G m_p}{r}}$$

$$v_{em} = \sqrt{\frac{2 G \frac{1}{10} m_e}{\frac{1}{2} r_e}}$$

$$v_{ee} = \sqrt{\frac{2 G m_e}{r_e}}$$

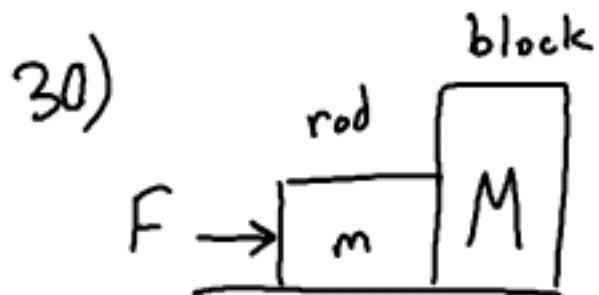
$$v_{em} = v_{ee} \sqrt{\frac{2}{10}}$$



$$f = \frac{1}{T} = \frac{1}{2\pi \sqrt{\frac{I}{mgd}}} = \frac{1}{2\pi} \sqrt{\frac{mgR}{\frac{1}{2}MR^2 + MR^2}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{\cancel{mgR}}{\frac{3}{2}\cancel{MR^2}}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{2g}{3R}} \quad \text{letter C}$$



block on rod =
rod on block;
3rd law

$$F_{\text{NET}} = ma$$

$$F = (m+M)a$$

$$\frac{F}{m+M} = a$$

block

$$F_b = M \left(\frac{F}{m+M} \right)$$

31)

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad \leftarrow \text{tells us, it's a spring oscillating}$$

$$F = -kx$$

$$ma = -kx$$

$$a = \frac{-kx}{m}$$

32)

$$U = -3x^2 + 2x \quad x \text{ when } F = 0$$

$$W = \int F dx \quad - \int F dx = -3x^2 + 2x$$

$$W = -\Delta U$$

$$\frac{d}{dx} \left(- \int F dx \right) = \frac{d}{dx} (-3x^2 + 2x)$$

$$-F = -6x + 2$$

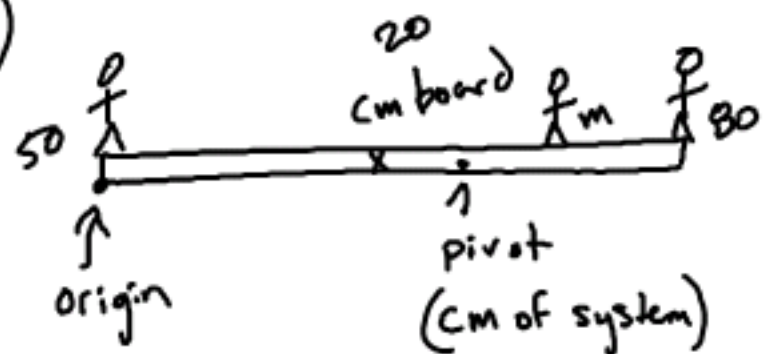
$$F = 6x - 2$$

Set $F = 0$ solve for x

$$0 = 6x - 2$$

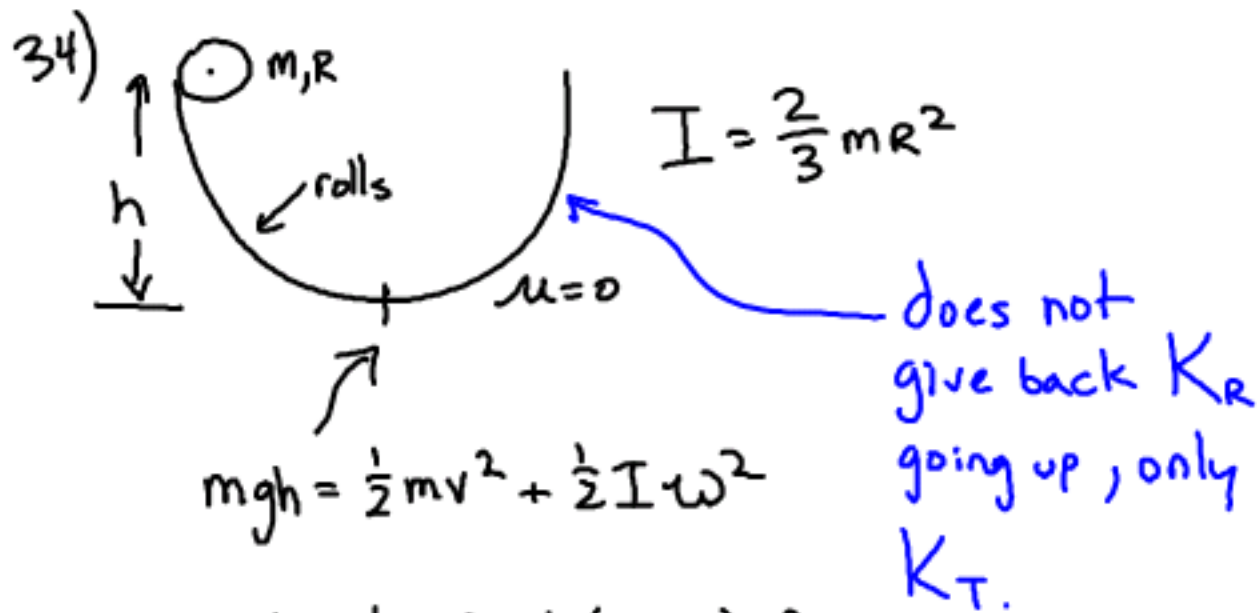
$$\frac{2}{6} = x = \frac{1}{3}$$

33)



$$X_{cm} = \frac{m_1 r_1 + m_2 r_2 \dots}{m_1 + m_2 + \dots}$$

$$5 = \frac{4(20) + 6(m) + 8(80)}{150 + m}$$



$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\frac{v^2}{R^2}$$

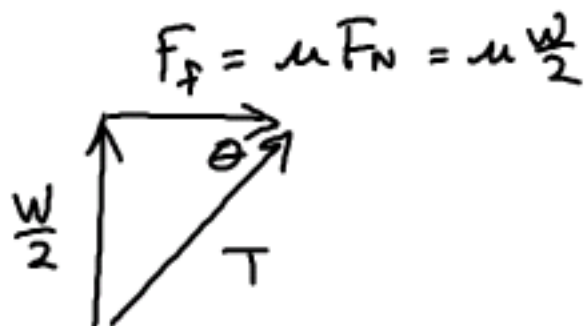
$$mgh = \frac{5}{6}mv^2$$

$$\frac{5}{3}\left(\frac{1}{2}mv^2\right)$$

$$mgh = \frac{5}{3}K$$

$$\frac{3}{5}mgh = K$$

35)



$$F_f = T \cos \theta \quad T \sin \theta = \frac{W}{2}$$

$$\mu \frac{W}{2} = \frac{W}{2} \cos \theta$$

$$T = \frac{W}{2 \sin \theta}$$

