Sample exam II solutions

1) F always points the same direction as a, hence as ΔV b/c $\vec{a} = \frac{\Delta \vec{V}}{t}$

E wrong b/c F and ΔV must be same way

2)

$$P = \frac{W}{t} \qquad W = \Delta K = \frac{1}{2}mv^2$$

$$P = \frac{\frac{1}{2}mv^2}{t} \qquad V_2 = \sqrt{2}\, V_1$$

or

$$\frac{\cancel{P_1} = \dfrac{\cancel{\frac{1}{2}}\cancel{m}v_1^2}{\cancel{t}}}{\cancel{P_1} = \dfrac{\cancel{\frac{1}{2}}\cancel{m}v_2^2}{2\cancel{t}}} \Rightarrow 1 = \frac{2V_1^2}{V_2^2}$$

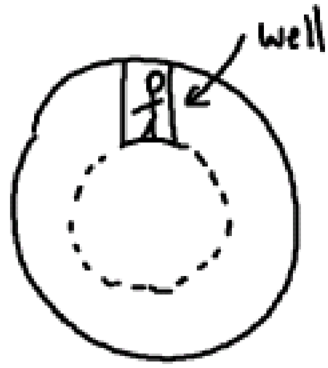
$$V_2^2 = 2V_1^2$$

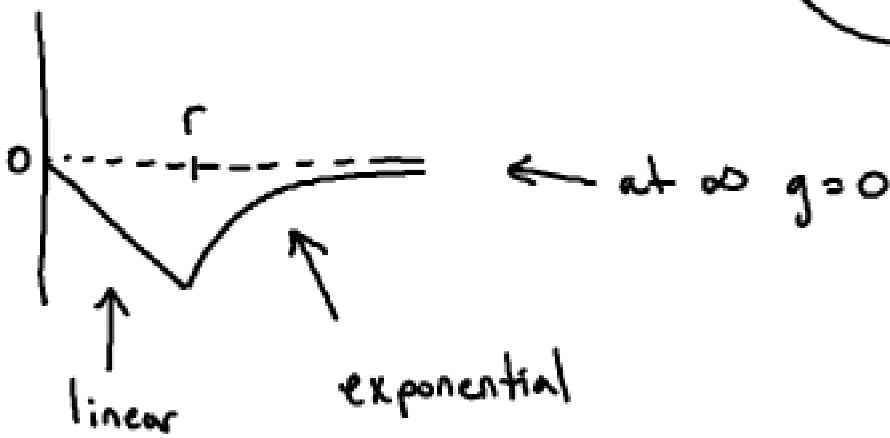
$$V_2 = \sqrt{2}\, V_1$$

3) g vs. r

assume + is away from earth

$$g = G\frac{m}{r^2}$$





4) **SHM**

$$x = A\cos\omega t$$

$$a = -A\omega^2\cos\omega t$$

$$a = -\omega^2 \underline{A\cos\omega t}$$

$$a = -\omega^2 x$$

match

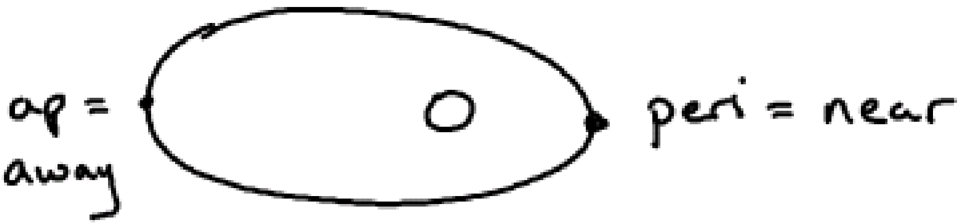
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Key parts

constant

negative

5)



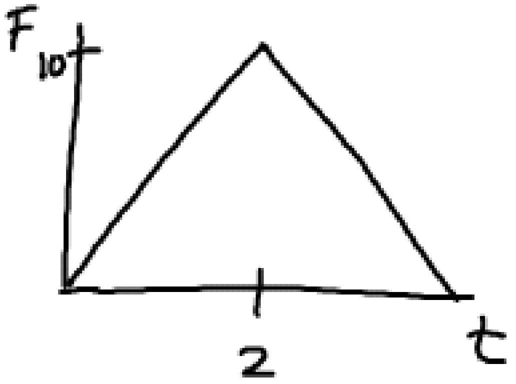
$$L_i = L_F$$

$$\cancel{m_1} v_1 r_{\perp 1} = \cancel{m_1} v_2 r_{\perp 2}$$

$$V_p \, 4.6 \times 10^{10} = V_A \cdot 7.0 \times 10^{10}$$

$$V_p = \frac{V_a \, 7.0 \times \cancel{10^{10}}}{4.6 \times \cancel{10^{10}}}$$

6)



F vs. t

$\int F\, dt = J = \Delta mv$

area = 20 units

$V_0 = 5$

$m = 1$

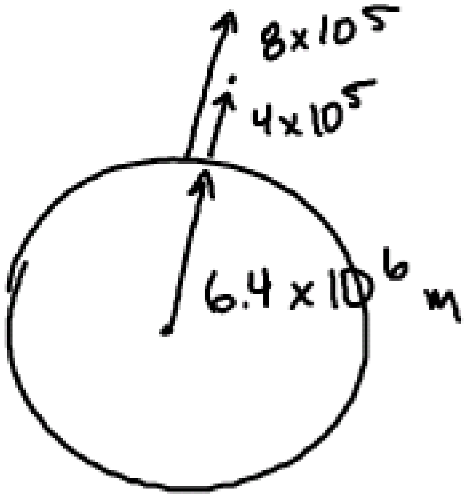
$20 = \Delta mv$

$20 = 1\,\Delta v$

$V_f = 25\ \frac{m}{s}$

$F \cdot t = \Delta mv$

7)



6.8
7.2

$$V = \sqrt{\frac{GM}{r}}$$

$$\frac{V_2}{V_0} = \frac{\sqrt{\frac{GM_e}{7.2 \times 10^6}}}{\sqrt{\frac{GM_e}{6.8 \times 10^6}}}$$

$$V_2 = V_0 \sqrt{\frac{6.8}{7.2}}$$

8)

$V = 10t - 2t^2$

$V_t = ?$ $\quad V_t$ @ $a = 0$ $\quad$ find a

$$\frac{dv}{dt} = a = \frac{d}{dt}\left(10t - 2t^2\right)$$

$a = 10 - 4t$ - set $a = 0$

find t

$0 = 10 - 4t$

$\frac{10}{4} = t = 2.5s$ - plug t into

V eq.

$V = 10t - 2t^2$

$V_t = 10(2.5) - 2(2.5)^2 = 12.5 \frac{m}{s}$

9)

$$a = 3x$$

$V = ?$ when $x = 5$

$$a = \frac{dv}{dt} = 3x$$

multiply by 1

$$\frac{dv}{dt} \cdot \left(\frac{dt}{dx} \cdot v\right) = 3x$$

$$\frac{dv}{dx} v = 3x \rightarrow \int v \, dv = \int 3x \, dx$$

$$\frac{v^2}{2} + \cancel{c}^{0} = \frac{3x^2}{2}$$

$$v^2 = 3x^2$$

$$V = \sqrt{3} x$$

(or →)

9) alternate solution

$$a = 3x$$

$$W = \int F\,dx = \Delta K$$

$$\int ma\,dx = \Delta K$$

$$\int m(3x)\,dx = \frac{1}{2}mv^2$$

$$\cancel{m}\frac{3x^2}{\cancel{2}} = \frac{1}{\cancel{2}}\cancel{m}v^2$$

$$\sqrt{3}(x) = v$$

10)

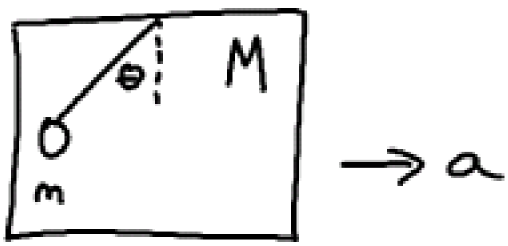
$$\Theta = 3t + 2t^2 \qquad V = r\omega$$

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}\left(3t + 2t^2\right) = 3 + 4t$$

$$\omega_5 = 3 + 4(5) = 23$$

$$V = (0.3)(23) = 6.9\ \frac{m}{s}$$

11)



$\Sigma F_y = m\cancel{a_y}^{\,0}$

$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$

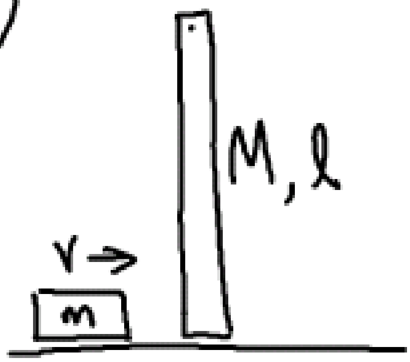
$\Sigma F_x = ma_x$

$T\sin\theta = ma$

$\frac{\cancel{m}g}{\cos\theta}\sin\theta = \cancel{m}a$

$g\tan\theta = a$

12)



inelastic collision

$L_i = L_F$ b/c of rotate

$L_i = L_F$

$mvr_\perp = I\omega$

$mvl = l^2\left(\frac{M}{3}+m\right)\omega$

$I_{TOT} = I_{rod} + I_{block}$

$= \frac{1}{3}Ml^2 + ml^2$

$= l^2\left(\frac{M}{3}+m\right)$

$$\frac{mv\cancel{l}}{l^{\cancel{2}}\left(\frac{M}{3}+m\right)} = \omega$$

$$\frac{3}{3}\,\frac{mv}{l\left(\frac{M}{3}+m\right)}$$

$$\frac{3mv}{l(M+3m)} = \omega$$

13)

$$I_{\parallel} = I_{cm} + md^2$$

$$= MR^2 + MR^2$$

$$= 2MR^2$$

14)

$$J = \int F\,dt = \Delta mv$$

$$-30 = \Delta mv = 5(V_F - 2)$$

$$\frac{-30}{5} = V_F - 2$$

$$-6 + 2 = V_F$$

$$-4 = V_F$$

opposite original direction

15)

$x = 20t + 30t^2 \leftarrow x = V_o t + \frac{1}{2}at^2$

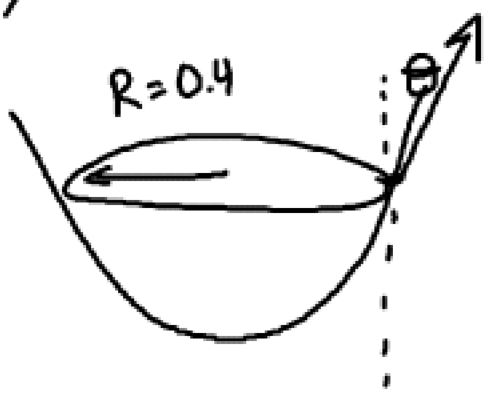
$y = 25 - 40t \leftarrow y = y_o + V_o t$

a in x direction only

$F = ma \quad a = 60$ b/c $\frac{1}{2}a = 30$ above

$F = 1(60) = 60\ N$

17)



<u>horizontal circle</u>

$$F_c = \Sigma F_r = m\frac{v^2}{r}$$

$$F_c = F_N\cos\theta$$

$$\Sigma F_y = 0$$

$$F_N\sin\theta = mg$$

$$F_N = \frac{mg}{\sin\theta}$$

$$F_c = \frac{mg}{\sin\theta}\cdot\cos\theta$$

$$m\frac{v^2}{r} = mg\cot\theta$$

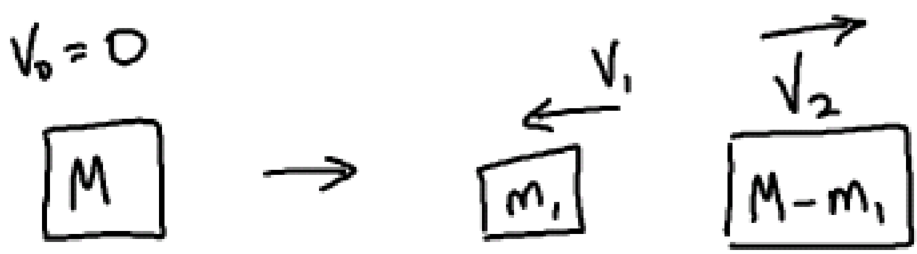
want ω

$\therefore$

$$\cancel{m}\omega^2 r = \cancel{m}g\cot\theta$$

$$\omega = \sqrt{\frac{g\cot\theta}{r}}$$

20)



$$K_1 = \frac{1}{2} m_1 v_1^2$$

$$P_i = P_F$$

$$0 = m_1 v_1 + (M - m_1) v_2$$

$$v_2 = \frac{-m_1 v_1}{(M - m_1)} \longrightarrow K_2 = \frac{1}{2}(M - m_1) v_2^2$$

$$K_2 = \frac{1}{2}(M - m_1)\left(\frac{-m_1 v_1}{M - m_1}\right)^2$$

$K_1 = \frac{1}{2} m_1 v_1^2$ given

$$K_2 = \frac{1}{2}\cancel{(M - m_1)}\left(\frac{m_1^2 v_1^2}{(M - m_1)^{\cancel{2}}}\right)$$

Sub in for $\frac{1}{2} m_1 v_1^2$ in eq. for K_2

$$K_2 = \frac{K_1 m_1}{(M - m_1)}$$

22)

$\tau_{net} = 0 \therefore \alpha = 0$

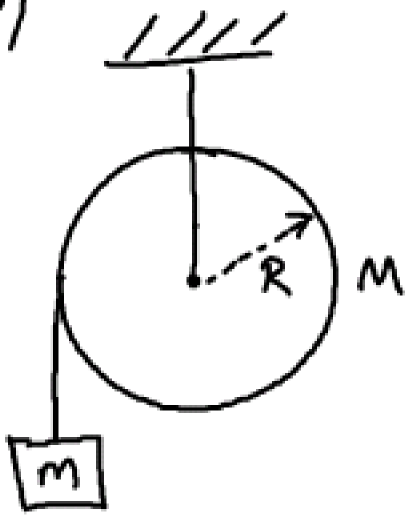
$F_{net} = ?$ $\Rightarrow$ not necessarily could be thru pivot

ω constant? $\Rightarrow$ not necessarily could be non-rigid object and I may change as it rotates $\therefore$

$I_1 \omega_1 = I_2 \omega_2$

Linear mom constant? – not necessarily, one object can have an impulse applied to it.

23)



block	pulley
$\Sigma F = ma$	$\Sigma \tau = I\alpha$
$mg - T = ma$	$rT = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$
$T = mg - ma$	$T = \frac{1}{2}ma$

$$\frac{1}{2}ma = mg - ma$$

$$\frac{3}{2}\cancel{m}a = \cancel{m}g$$

$$a = \frac{2}{3}g$$

24)

note: the center of mass never moves unless an outside force acts.

$p_i = p_F$

$0 = m_2 v_2 + m_1 v_1$

$-m_1 v_1 = m_2 v_2$

$\frac{v_1}{v_2} = -\frac{m_2}{m_1}$

25)

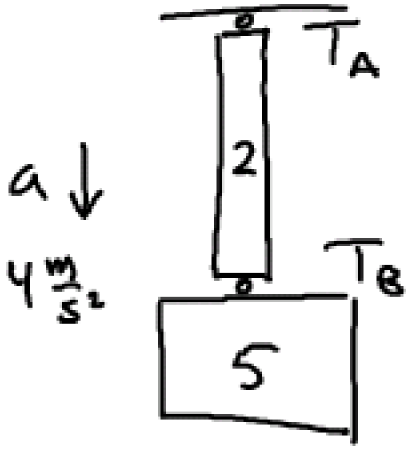
$$T_s = 2\pi\sqrt{\frac{m}{k}}$$ use m_{eff}

$$m_{eff} = \frac{m_1 \cdot m_2}{m_1 + m_2}$$

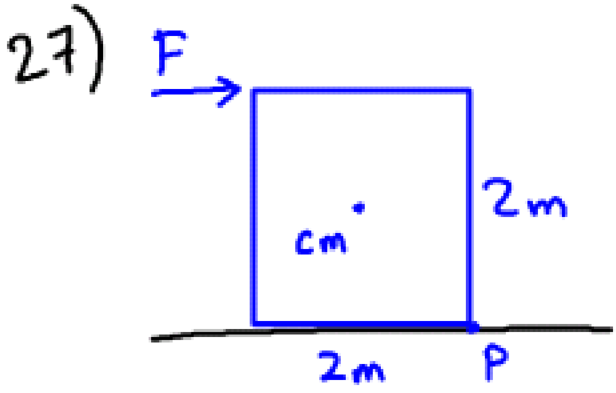
$$T_s = 2\pi\sqrt{\frac{\frac{m_1 \cdot m_2}{m_1 + m_2}}{k}}$$

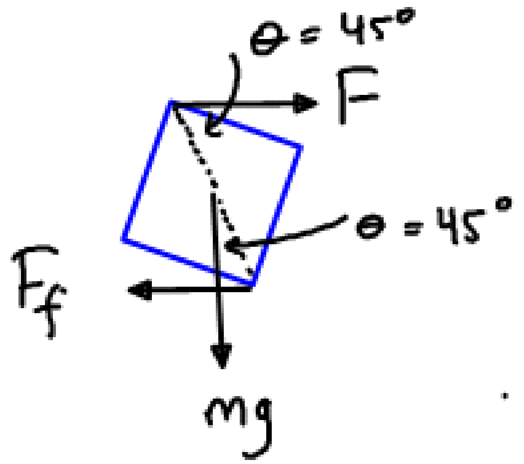
$$= 2\pi\sqrt{\frac{m_1 \cdot m_2}{k(m_1 + m_2)}}$$

26)



27)





$\Sigma \tau = I\alpha$ let $\alpha = 0$

use θ from first starting

$r\,mg\sin\theta - 2r\,F\sin\theta = 0$

$\cancel{r}\,mg\,\cancel{\sin\theta} = 2\,\cancel{r}\,F\,\cancel{\sin\theta}$

$$\frac{mg}{2} = F$$

28)

$r_m = \frac{1}{2} r_e$

$m_m = \frac{1}{10} m_e$

$V_{em} = ?$

$$-G\frac{m_1 m_2}{r} + \frac{1}{2} m v^2 = 0$$

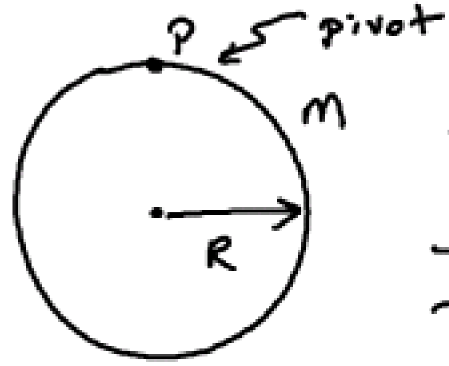
$$G\frac{\cancel{m_1} m_2}{r} = \frac{1}{2}\cancel{m} v^2$$

$$V_{esc} = \sqrt{\frac{2Gm_p}{r}}$$

$$\frac{V_{em} = \sqrt{\dfrac{2G\frac{1}{10}m_e}{\frac{1}{2}r_e}}}{V_{ee} = \sqrt{\dfrac{2Gm_e}{r_e}}}$$

$$V_{em} = V_{ee}\sqrt{\frac{2}{10}}$$

29) disk



$$I = ?$$

$$I_{cm} = \frac{1}{2} MR^2$$

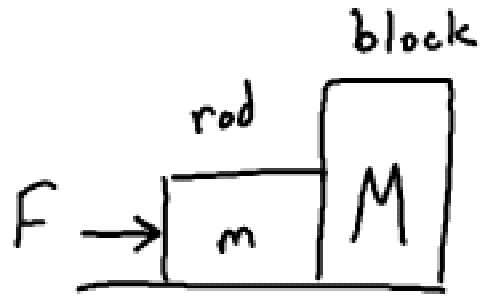
$$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{I}{mgd}}} = \frac{1}{2\pi}\sqrt{\frac{mgR}{\frac{1}{2}MR^2 + MR^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{\cancel{m}g\cancel{R}}{\frac{3}{2}\cancel{M}R^{\cancel{2}}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2g}{3R}}$$

letter C

30)



block on rod = rod on block; 3rd law

$F_{NET} = ma$

$$F = (m+M)a$$

$$\frac{F}{m+M} = a$$

<u>block</u>

$$F_b = M\left(\frac{F}{m+M}\right)$$

31)

$$E = \frac{1}{2}mv^2 + \frac{1}{2}ks^2$$ ← tells us, it's a spring oscillating

$$F = -kx$$

$$ma = -kx$$

$$a = \frac{-kx}{m}$$

32)

$U = -3x^2 + 2x$ $\quad$ x when $F = 0$

$W = \int F\,dx$

$-\int F\,dx = -3x^2 + 2x$

$W = -\Delta U$

$$\frac{d}{dx}\left(-\int F\,dx\right) = \frac{d}{dx}\left(-3x^2 + 2x\right)$$

$$-F = -6x + 2$$

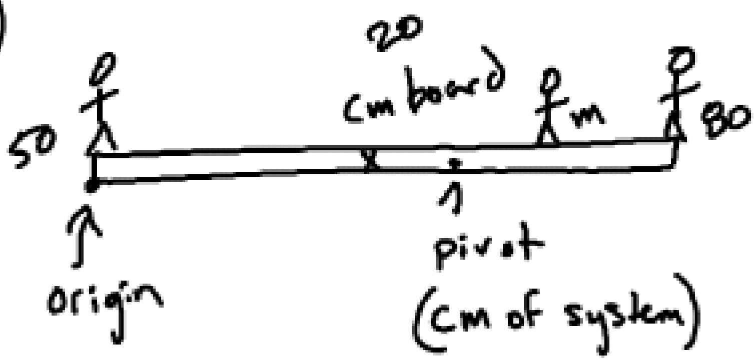
$$F = 6x - 2$$

Set $F = 0$ solve for x

$$0 = 6x - 2$$

$$\frac{2}{6} = x = \frac{1}{3}$$

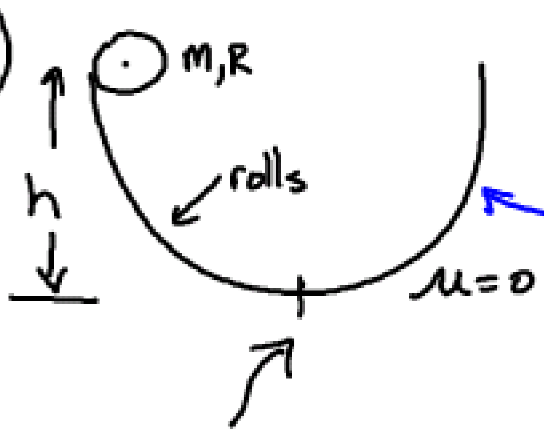
33)



$$X_{cm} = \frac{m_1 r_1 + m_2 r_2 \ldots}{m_1 + m_2 + \ldots}$$

$$5 = \frac{4(20) + 6(m) + 8(80)}{150 + m}$$

34)



$I = \frac{2}{3} m R^2$

does not give back K_R going up, only K_T.

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{2}{3} m R^2\right)\frac{v^2}{R^2}$$

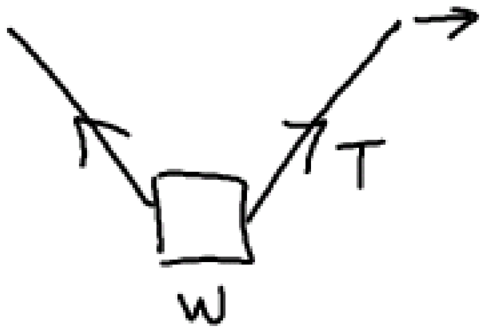
$$mgh = \frac{5}{6} m v^2$$

$$\frac{5}{3}\left(\frac{1}{2} m v^2\right)$$

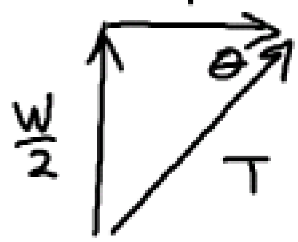
$$mgh = \frac{5}{3} K$$

$$\frac{3}{5} mgh = K$$

35)



$F_f = \mu F_N = \mu \frac{W}{2}$



$F_f = T\cos\theta \quad T\sin\theta = \frac{W}{2}$

$T = \frac{W}{2\sin\theta}$

$\mu \frac{\cancel{W}}{\cancel{2}} = \frac{\cancel{W}}{\cancel{2}\sin\theta}\cos\theta$