

Subatomic Physics

Problem A**BINDING ENERGY****PROBLEM**

Each egg of *Caraphractus cinctus*, a parasitic wasp, has a mass of about 2.0×10^{-10} kg. Consider the formation of ${}^{16}_8\text{O}$ from H atoms and neutrons. How many nuclei of ${}^{16}_8\text{O}$ must be formed to produce a mass defect equal to the mass of one *Caraphractus cinctus* egg? What is the total binding energy of these ${}^{16}_8\text{O}$ nuclei? The atomic mass of ${}^{16}_8\text{O}$ is 15.994 915 u.

SOLUTION**1. DEFINE**

Given: $m_{\text{egg}} = 2.0 \times 10^{-10}$ kg
 element formed = ${}^{16}_8\text{O}$
 $Z = 8$ $N = 16 - 8 = 8$
 atomic mass of ${}^{16}_8\text{O} = 15.994\ 915$ u
 atomic mass of H = 1.007 825 u
 $m_n = 1.008\ 665$ u

Unknown: $\Delta m = ?$
 $n =$ number of ${}^{16}_8\text{O}$ nuclei formed = ?
 total $E_{\text{bind}} = ?$

2. PLAN

Choose the equation(s) or situation: First find the mass defect using the equation for mass defect.

$$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass}$$

To determine the number of oxygen-16 nuclei that must be formed to produce a total mass defect equal to the mass of a *Caraphractus cinctus* egg, calculate the ratio of the mass of one egg to the mass defect.

$$\text{the number of } {}^{16}_8\text{O nuclei formed} = n = \frac{m_{\text{egg}}}{\Delta m}$$

Finally, to find the total binding energy of all ${}^{16}_8\text{O}$ nuclei, use the equation for the binding energy of a nucleus and multiply it by n .

$$\begin{aligned} \text{total } E_{\text{bind}} &= nE_{\text{bind}} = n\Delta mc^2 \\ \text{total } E_{\text{bind}} &= n\Delta m (931.49 \text{ MeV/u}) \end{aligned}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\begin{aligned} \Delta m &= 8(1.007\ 825 \text{ u}) + 8(1.008\ 665 \text{ u}) - 15.994\ 915 \text{ u} \\ \Delta m &= 8.062\ 600 \text{ u} + 8.069\ 320 \text{ u} - 15.994\ 915 \text{ u} \\ \Delta m &= 0.137\ 005 \text{ u} \end{aligned}$$

$$n = \frac{(2.0 \times 10^{-10} \text{ kg})}{(0.137\ 005 \text{ u})} \left(\frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}} \right) = \boxed{8.8 \times 10^{17} \text{ nuclei}}$$

$$\text{total } E_{\text{bind}} = (8.8 \times 10^{17})(0.137\ 005 \text{ u})(931.49 \text{ MeV/u})$$

$$\text{total } E_{\text{bind}} = \boxed{1.1 \times 10^{20} \text{ MeV}}$$

4. EVALUATE

For every nucleus of ${}^{16}_8\text{O}$ formed, the mass defect is 0.137 005 u, and the mass of the nucleus formed is 15.994 915 u. When Δm has a total value of 2.0×10^{-10} kg, 8.8×10^{17} nuclei, or 2.3×10^{-8} kg of ${}^{16}_8\text{O}$ will have formed.

ADDITIONAL PRACTICE

- In 1993, the United States had more than 100 operational nuclear reactors producing about 30 percent of the world's nuclear energy, or 610 TW•h.
 - Find the mass defect corresponding to a binding energy equal to that energy output.
 - How many ${}^2_1\text{H}$ nuclei would be needed to provide this mass defect?
 - How many ${}^{56}_{26}\text{Fe}$ nuclei would be needed to provide this mass defect?
 - How many ${}^{226}_{88}\text{Ra}$ nuclei would be needed to provide this mass defect?
- In 1976, Montreal hosted the Summer Olympics. To complete the new velodrome, the 4.1×10^7 kg roof had to be raised 10.0 cm to be placed in the exact position.
 - Find the energy needed to raise the roof.
 - Find the mass of ${}^{56}_{26}\text{Fe}$ that is formed when an amount of energy equal to that calculated in part (a) is obtained from binding H atoms and neutrons in iron-56 nuclei.
- Nuclear-energy production worldwide was 2.0×10^3 TW•h in 1993. What mass of ${}^{235}_{92}\text{U}$ releases an equivalent amount of energy in the form of binding energy?
- In 1993, the United States burned about 2.00×10^8 kg of coal to produce about 2.1×10^{19} J of energy. Suppose that instead of *burning* coal, you obtain energy by *forming* coal (${}^{12}_6\text{C}$) out of H atoms and neutrons. What amount of coal must be formed to provide 2.1×10^{19} J of energy? Assume 100 percent efficiency.
- The sun radiates energy at a rate of 3.9×10^{26} J/s. Suppose that all the sun's energy occurs because of the formation of ${}^4_2\text{He}$ from H atoms and neutrons. Find the number of reactions that take place each second.
- Sulzer Brothers, a Swiss company, made powerful diesel engines for the container ships built for American President Lines. The power of each 12-cylinder engine is about 42 MW. Suppose the turbines use the formation of ${}^{14}_7\text{N}$ for the energy-releasing process. What mass of nitrogen would have to be formed to provide enough energy for 24 h of continuous work? Assume the turbines are 100 percent efficient.
- A hundred years ago, the most powerful hydroelectric plant in the world produced 3.84×10^7 W of electric power. Find the total mass of ${}^{12}_6\text{C}$ atoms that must be formed each second from H atoms and neutrons to produce the same power output.

Subatomic Physics

Additional Practice A

Givens

1. $E = 610 \text{ TW}\cdot\text{h}$

atomic mass of ${}^2_1\text{H} = 2.014 \text{ 102 u}$

atomic mass of ${}^{56}_{26}\text{Fe} = 55.934 \text{ 940 u}$

atomic mass of ${}^{226}_{88}\text{Ra} = 226.025 \text{ 402 u}$

Solutions

a. $\Delta m = \frac{E}{c^2} = \frac{(610 \times 10^9 \text{ kW}\cdot\text{h})(3.6 \times 10^6 \text{ J/kW}\cdot\text{h})}{(3.00 \times 10^8 \text{ m/s})^2}$

$\Delta m = \boxed{24 \text{ kg}}$

b. $n = \frac{\Delta m}{\text{atomic mass of } {}^2_1\text{H}} = \frac{24 \text{ kg}}{(1.66 \times 10^{-27} \text{ kg/u})(2.014 \text{ 102 u})}$

$n = \boxed{7.2 \times 10^{27} \text{ } {}^2_1\text{H nuclei}}$

c. $n = \frac{\Delta m}{\text{atomic mass of } {}^{56}_{26}\text{Fe}} = \frac{24 \text{ kg}}{(1.66 \times 10^{-27} \text{ kg/u})(55.934 \text{ 940 u})}$

$n = \boxed{2.6 \times 10^{26} \text{ } {}^{56}_{26}\text{Fe nuclei}}$

d. $n = \frac{\Delta m}{\text{atomic mass of } {}^{226}_{88}\text{Ra}} = \frac{24 \text{ kg}}{(1.66 \times 10^{-27} \text{ kg/u})(226.025 \text{ 402 u})}$

$n = \boxed{6.4 \times 10^{25} \text{ } {}^{226}_{88}\text{Ra nuclei}}$

2. $m = 4.1 \times 10^7 \text{ kg}$

$h = 10.0 \text{ cm}$

$Z = 26$

$N = 56 - 26 = 30$

$m_H = 1.007 \text{ 825 u}$

$m_n = 1.008 \text{ 665 u}$

atomic mass of ${}^{56}_{26}\text{Fe} = 55.934 \text{ 940 u}$

a. $E = mgh = (4.1 \times 10^7 \text{ kg})(9.81 \text{ m/s}^2)(0.100 \text{ m})$

$E = \boxed{4.0 \times 10^7 \text{ J}}$

b. $E_{tot} = \frac{(4.0 \times 10^7 \text{ J})(1 \times 10^{-6} \text{ MeV/eV})}{(1.60 \times 10^{-19} \text{ J/eV})} = 2.5 \times 10^{20} \text{ MeV}$

$\Delta m_{tot} = \frac{2.5 \times 10^{20} \text{ MeV}}{931.49 \text{ MeV/u}} = 2.7 \times 10^{17} \text{ u}$

$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } {}^{56}_{26}\text{Fe}$

$\Delta m = 26(1.007 \text{ 825 u}) + 30(1.008 \text{ 665 u}) - 55.934 \text{ 940 u}$

$\Delta m = 0.528 \text{ 460 u}$

$E_{bind} = (0.528 \text{ 460 u})\left(931.49 \frac{\text{MeV}}{\text{u}}\right)$

$E_{bind} = 492.26 \text{ MeV}$

$n = \frac{E_{tot}}{E_{bind}} = \frac{2.5 \times 10^{20} \text{ MeV}}{492.26 \text{ MeV}} = 5.1 \times 10^{17} \text{ reactions}$

$m_{tot} = (5.1 \times 10^{17})(55.934 \text{ 940 u})\left(1.66 \times 10^{-27} \frac{\text{kg}}{\text{u}}\right) = \boxed{4.7 \times 10^{-8} \text{ kg}}$

Givens

3. $E = 2.0 \times 10^3 \text{ TW} \cdot \text{h} = 2.0 \times 10^{15} \text{ W} \cdot \text{h}$
 atomic mass of ${}_{92}^{235}\text{U} = 235.043 \text{ 924 u}$
 atomic mass of H = 1.007 825 u
 $m_n = 1.008 \text{ 665 u}$
 $Z = 92$
 $N = 235 - 92 = 143$

Solutions

$$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } {}_{92}^{235}\text{U}$$

$$\Delta m = 92(1.007 \text{ 825 u}) + 143(1.008 \text{ 665 u}) - 235.043 \text{ 924 u} = 1.915 \text{ 071 u}$$

$$E_{\text{bind}} = (1.915 \text{ 071 u}) \left(931.49 \frac{\text{MeV}}{\text{u}} \right) = 1.7839 \times 10^3 \text{ MeV} = 1.7839 \times 10^9 \text{ eV}$$

$$E_{\text{bind}} = (1.7839 \times 10^9 \text{ eV}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right) = 2.85 \times 10^{-10} \text{ J}$$

$$E = (2.0 \times 10^{15} \text{ W} \cdot \text{h}) \left(\frac{3.60 \times 10^3 \text{ s}}{\text{h}} \right) = 7.2 \times 10^{18} \text{ J}$$

$$n = \frac{E}{E_{\text{bind}}} = \frac{7.2 \times 10^{18} \text{ J}}{2.85 \times 10^{-10} \text{ J}} = 2.5 \times 10^{28} \text{ reactions}$$

$$m_{\text{tot}} = (2.5 \times 10^{28})(235.043 \text{ 924 u}) \left(1.66 \times 10^{-27} \frac{\text{kg}}{\text{u}} \right) = \boxed{9.8 \times 10^3 \text{ kg}}$$

4. $E = 2.1 \times 10^{19} \text{ J}$
 atomic mass of ${}_{6}^{12}\text{C} = 12.000 \text{ 000 u}$
 atomic mass of H = 1.007 825 u
 $m_n = 1.008 \text{ 665 u}$
 $Z = 6$
 $N = 12 - 6 = 6$

$$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } {}_{6}^{12}\text{C}$$

$$\Delta m = 6(1.007 \text{ 825 u}) + 6(1.008 \text{ 665 u}) - 12.000 \text{ 000 u}$$

$$\Delta m = 9.8940 \times 10^{-2} \text{ u}$$

$$E_{\text{bind}} = (9.8940 \times 10^{-2} \text{ u}) \left(931.49 \frac{\text{MeV}}{\text{u}} \right) = 92.162 \text{ MeV}$$

$$E_{\text{bind}} = (92.162 \times 10^6 \text{ eV}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right) = 1.47 \times 10^{-11} \text{ J}$$

$$n = \frac{E}{E_{\text{bind}}} = \frac{2.1 \times 10^{19} \text{ J}}{1.47 \times 10^{-11} \text{ J}} = 1.4 \times 10^{30} \text{ reactions}$$

$$m_{\text{tot}} = (1.4 \times 10^{30})(12.000 \text{ 000 u}) \left(1.66 \times 10^{-27} \frac{\text{kg}}{\text{u}} \right) = \boxed{2.8 \times 10^4 \text{ kg}}$$

5. $P_{\text{tot}} = 3.9 \times 10^{26} \text{ J/s}$
 $Z = 2$
 $N = 4 - 2 = 2$
 atomic mass of ${}_{2}^4\text{He} = 4.002 \text{ 602 u}$
 atomic mass of H = 1.007 825 u
 $m_n = 1.008 \text{ 665 u}$

$$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } {}_{2}^4\text{He}$$

$$\Delta m = (2)(1.007 \text{ 825 u}) + (2)(1.008 \text{ 665 u}) - 4.002 \text{ 602 u}$$

$$\Delta m = 0.030 \text{ 378 u}$$

$$E = (0.030 \text{ 378 u}) \left(931.49 \frac{\text{MeV}}{\text{u}} \right)$$

$$E = 28.297 \text{ MeV}$$

$$\frac{n}{\Delta t} = \frac{P_{\text{tot}}}{E} = \frac{(3.9 \times 10^{26} \text{ J/s})(1 \times 10^{-6} \text{ MeV/eV})}{(1.60 \times 10^{-19} \text{ J/eV})(28.297 \text{ MeV})}$$

$$\frac{n}{\Delta t} = \boxed{8.6 \times 10^{37} \text{ reactions/s}}$$

6. $P = 42 \text{ MW} = 42 \times 10^6 \text{ W}$
 atomic mass of ${}_{7}^{14}\text{N} = 14.003 \text{ 074 u}$
 atomic mass of H = 1.007 825 u
 $m_n = 1.008 \text{ 665 u}$
 $Z = 7$
 $N = 14 - 7 = 7$
 $\Delta t = 24 \text{ h}$

$$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } {}_{7}^{14}\text{N}$$

$$\Delta m = 7(1.007 \text{ 825 u}) + 7(1.008 \text{ 665 u}) - 14.003 \text{ 074 u}$$

$$\Delta m = 0.112 \text{ 356 u}$$

$$E_{\text{bind}} = (0.112 \text{ 356 u}) \left(931.49 \frac{\text{MeV}}{\text{u}} \right) = 104.66 \text{ MeV}$$

$$E_{\text{bind}} = (104.66 \times 10^6 \text{ eV}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right) = 1.67 \times 10^{-11} \text{ J}$$

$$n = \frac{P\Delta t}{E_{\text{bind}}} = \frac{(42 \times 10^6 \text{ W})(24 \text{ h})(3600 \text{ s/h})}{1.67 \times 10^{-11} \text{ J}} = 2.2 \times 10^{23} \text{ reactions}$$

$$m_{\text{tot}} = (2.2 \times 10^{23})(14.003 \text{ 074 u}) \left(1.66 \times 10^{-27} \frac{\text{kg}}{\text{u}} \right) = \boxed{5.1 \times 10^{-3} \text{ kg} = 5.1 \text{ g}}$$

Givens

7. $P = 3.84 \times 10^7 \text{ W}$
 atomic mass of $^{12}_6\text{C} = 12.000\,000 \text{ u}$
 atomic mass of H = $1.007\,825 \text{ u}$
 $m_n = 1.008\,665 \text{ u}$
 $Z = 6$
 $N = 12 - 6 = 6$

Solutions

$$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } ^{12}_6\text{C}$$

$$\Delta m = 6(1.007\,825 \text{ u}) + 6(1.008\,665 \text{ u}) - 12.000\,000 \text{ u}$$

$$\Delta m = 9.8940 \times 10^{-2} \text{ u}$$

$$E_{\text{bind}} = (9.8940 \times 10^{-2} \text{ u}) \left(931.49 \times 10^6 \frac{\text{eV}}{\text{u}} \right) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right)$$

$$E_{\text{bind}} = 1.47 \times 10^{-11} \text{ J}$$

$$\frac{n}{\Delta t} = \frac{P}{E_{\text{bind}}} = \frac{3.84 \times 10^7 \text{ W}}{1.47 \times 10^{-11} \text{ J}} = 2.61 \times 10^{18} \text{ reactions/s}$$

$$\frac{m_{\text{tot}}}{\Delta t} = (2.61 \times 10^{18} \text{ s}^{-1})(12.000\,000 \text{ u}) \left(1.66 \times 10^{-27} \frac{\text{kg}}{\text{u}} \right) = \boxed{5.20 \times 10^{-8} \text{ kg/s}}$$

Additional Practice B

1. $^{238}_{92}\text{U} + {}^1_0n \rightarrow X$

$$X \rightarrow {}^{939}_{93}\text{Np} + {}^0_{-1}e + \bar{\nu}$$

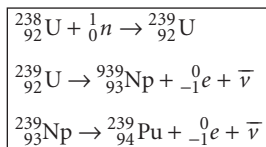
$${}^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + {}^0_{-1}e + \bar{\nu}$$

mass number of X = $238 + 1 = 239$

atomic number of X = $92 + 0 = 92$ (uranium)

$$X = {}^{239}_{92}\text{U}$$

The equations are as follows:



2. $X \rightarrow Y + {}^4_2\text{He}$

$$Y \rightarrow Z + {}^4_2\text{He}$$

$$Z \rightarrow {}^{212}_{83}\text{Bi} + {}^0_{-1}e + \bar{\nu}$$

mass number of Z = $212 + 0 = 212$

atomic number of Z = $83 - 1 = 82$ (lead)

$$Z = {}^{212}_{82}\text{Pb}$$

mass number of Y = $212 + 4 = 216$

atomic number of Y = $82 + 2 = 84$ (polonium)

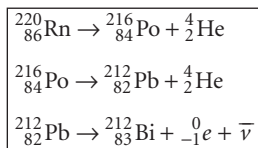
$$Y = {}^{216}_{84}\text{Po}$$

mass number of X = $216 + 4 = 220$

atomic number of X = $84 + 2 = 86$ (radon)

$$X = {}^{220}_{86}\text{Rn}$$

The equations are as follows:



3. $X \rightarrow {}^{135}_{56}\text{Ba} + {}^0_{-1}e + \bar{\nu}$

mass number of X = $135 + 0 = 135$

atomic number of X = $56 + (-1) = 55$ (cesium)

$$X = {}^{135}_{55}\text{Cs}$$

