NAME ________________ DATE ________ CLASS ________

Subatomic Physics

Problem A

BINDING ENERGY

PROBLEM

Each egg of *Caraphractus cinctus*, a parasitic wasp, has a mass of about 2.0×10^{-10} kg. Consider the formation of $^{16}_{8}O$ from H atoms and neutrons. How many nuclei of $^{16}_{8}O$ must be formed to produce a mass defect equal to the mass of one *Caraphractus cinctus* egg? What is the total binding energy of these $^{16}_{8}O$ nuclei? The atomic mass of $^{16}_{8}O$ is 15.994 915 u.

SOLUTION

1. DEFINE

Given:

$m_{egg} = 2.0 \times 10^{-10}$ kg

element formed = $^{16}_{8}O$

$Z = 8 \qquad N = 16 - 8 = 8$

atomic mass of $^{16}_{8}O$ = 15.994 915 u

atomic mass of H = 1.007 825 u

$m_n = 1.008\ 665$ u

Unknown:

$\Delta m = ?$

n = number of $^{16}_{8}O$ nuclei formed = ?

total E_{bind} = ?

2. PLAN

Choose the equation(s) or situation: First find the mass defect using the equation for mass defect.

$$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass}$$

To determine the number of oxygen-16 nuclei that must be formed to produce a total mass defect equal to the mass of a *Caraphractus cinctus* egg, calculate the ratio of the mass of one egg to the mass defect.

$$\text{the number of } ^{16}_{8}\text{O nuclei formed} = n = \frac{m_{egg}}{\Delta m}$$

Finally, to find the total binding energy of all $^{16}_{8}O$ nuclei, use the equation for the binding energy of a nucleus and multiply it by n.

$$\text{total } E_{bind} = nE_{bind} = n\Delta mc^2$$

$$\text{total } E_{bind} = n\Delta m(931.49 \text{ MeV/u})$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\Delta m = 8(1.007\ 825 \text{ u}) + 8(1.008\ 665 \text{ u}) - 15.994\ 915 \text{ u}$$

$$\Delta m = 8.062\ 600 \text{ u} + 8.069\ 320 \text{ u} - 15.994\ 915 \text{ u}$$

$$\Delta m = 0.137\ 005 \text{ u}$$

$$n = \frac{(2.0 \times 10^{-10} \text{ kg})}{(0.137\ 005 \text{ u})}\left(\frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}}\right) = \boxed{8.8 \times 10^{17} \text{ nuclei}}$$

$$\text{total } E_{bind} = (8.8 \times 10^{17})(0.137\ 005 \text{ u})(931.49 \text{ MeV/u})$$

$$\text{total } E_{bind} = \boxed{1.1 \times 10^{20} \text{ MeV}}$$

4. EVALUATE For every nucleus of $^{16}_{8}O$ formed, the mass defect is 0.137 005 u, and the mass of the nucleus formed is 15.994 915 u. When Δm has a total value of 2.0×10^{-10} kg, 8.8×10^{17} nuclei, or 2.3×10^{-8} kg of $^{16}_{8}O$ will have formed.

ADDITIONAL PRACTICE

1. In 1993, the United States had more than 100 operational nuclear reactors producing about 30 percent of the world's nuclear energy, or 610 TW•h.

- **a.** Find the mass defect corresponding to a binding energy equal to that energy output.
- **b.** How many $^{2}_{1}H$ nuclei would be needed to provide this mass defect?
- **c.** How many $^{56}_{26}Fe$ nuclei would be needed to provide this mass defect?
- **d.** How many $^{226}_{88}Ra$ nuclei would be needed to provide this mass defect?

2. In 1976, Montreal hosted the Summer Olympics. To complete the new velodrome, the 4.1×10^{7} kg roof had to be raised 10.0 cm to be placed in the exact position.

- **a.** Find the energy needed to raise the roof.
- **b.** Find the mass of $^{56}_{26}Fe$ that is formed when an amount of energy equal to that calculated in part (a) is obtained from binding H atoms and neutrons in iron-56 nuclei.

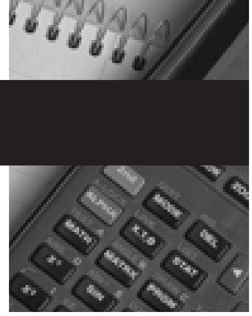
3. Nuclear-energy production worldwide was 2.0×10^{3} TW•h in 1993. What mass of $^{235}_{92}U$ releases an equivalent amount of energy in the form of binding energy?

4. In 1993, the United States burned about 2.00×10^{8} kg of coal to produce about 2.1×10^{19} J of energy. Suppose that instead of *burning* coal, you obtain energy by *forming* coal ($^{12}_{6}C$) out of H atoms and neutrons. What amount of coal must be formed to provide 2.1×10^{19} J of energy? Assume 100 percent efficiency.

5. The sun radiates energy at a rate of 3.9×10^{26} J/s. Suppose that all the sun's energy occurs because of the formation of $^{4}_{2}He$ from H atoms and neutrons. Find the number of reactions that take place each second.

6. Sulzer Brothers, a Swiss company, made powerful diesel engines for the container ships built for American President Lines. The power of each 12-cylinder engine is about 42 MW. Suppose the turbines use the formation of $^{14}_{7}N$ for the energy-releasing process. What mass of nitrogen would have to be formed to provide enough energy for 24 h of continuous work? Assume the turbines are 100 percent efficient.

7. A hundred years ago, the most powerful hydroelectric plant in the world produced 3.84×10^{7} W of electric power. Find the total mass of $^{12}_{6}C$ atoms that must be formed each second from H atoms and neutrons to produce the same power output.



Subatomic Physics

Additional Practice A

Givens

1. $E = 610\text{ TW}\bullet\text{h}$

Solutions

a. $\Delta m = \dfrac{E}{c^2} = \dfrac{(610 \times 10^9\text{ kW}\bullet\text{h})(3.6 \times 10^6\text{ J/kW}\bullet\text{h})}{(3.00 \times 10^8\text{ m/s})^2}$

$\Delta m = \boxed{24\text{ kg}}$

Givens: atomic mass of ${}^2_1\text{H} =$ 2.014 102 u

b. $n = \dfrac{\Delta m}{\text{atomic mass of } {}^2_1\text{H}} = \dfrac{24\text{ kg}}{(1.66 \times 10^{-27}\text{ kg/u})(2.014\ 102\text{ u})}$

$n = \boxed{7.2 \times 10^{27}\ {}^2_1\text{H nuclei}}$

Givens: atomic mass of ${}^{56}_{26}\text{Fe} =$ 55.934 940 u

c. $n = \dfrac{\Delta m}{\text{atomic mass of } {}^{56}_{26}\text{Fe}} = \dfrac{24\text{ kg}}{(1.66 \times 10^{-27}\text{ kg/u})(55.934\ 940\text{ u})}$

$n = \boxed{2.6 \times 10^{26}\ {}^{56}_{26}\text{Fe nuclei}}$

Givens: atomic mass of ${}^{226}_{88}\text{Ra} =$ 226.025 402 u

d. $n = \dfrac{\Delta m}{\text{atomic mass of } {}^{226}_{88}\text{Ra}} = \dfrac{24\text{ kg}}{(1.66 \times 10^{-27}\text{ kg/u})(226.025\ 402\text{ u})}$

$n = \boxed{6.4 \times 10^{25}\ {}^{226}_{88}\text{Ra nuclei}}$

Givens

2. $m = 4.1 \times 10^7\text{ kg}$

$h = 10.0\text{ cm}$

$Z = 26$

$N = 56 - 26 = 30$

$m_H = 1.007\ 825\text{ u}$

$m_n = 1.008\ 665\text{ u}$

atomic mass of ${}^{56}_{26}\text{Fe} =$ 55.934 940 u

Solutions

a. $E = mgh = (4.1 \times 10^7\text{ kg})(9.81\text{ m/s}^2)(0.100\text{ m})$

$E = \boxed{4.0 \times 10^7\text{ J}}$

b. $E_{tot} = \dfrac{(4.0 \times 10^7\text{ J})(1 \times 10^{-6}\text{ MeV/eV})}{(1.60 \times 10^{-19}\text{ J/eV})} = 2.5 \times 10^{20}\text{ MeV}$

$\Delta m_{tot} = \dfrac{2.5 \times 10^{20}\text{ MeV}}{931.49\text{ MeV/u}} = 2.7 \times 10^{17}\text{ u}$

$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } {}^{56}_{26}\text{Fe}$

$\Delta m = 26(1.007\ 825\text{ u}) + 30(1.008\ 665\text{ u}) - 55.934\ 940\text{ u}$

$\Delta m = 0.528\ 460\text{ u}$

$E_{bind} = (0.528\ 460\text{ u})\left(931.49\dfrac{\text{MeV}}{\text{u}}\right)$

$E_{bind} = 492.26\text{ MeV}$

$n = \dfrac{E_{tot}}{E_{bind}} = \dfrac{2.5 \times 10^{20}\text{ MeV}}{492.26\text{ MeV}} = 5.1 \times 10^{17}\text{ reactions}$

$m_{tot} = (5.1 \times 10^{17})(55.934\ 940\text{ u})\left(1.66 \times 10^{-27}\dfrac{\text{kg}}{\text{u}}\right) = \boxed{4.7 \times 10^{-8}\text{ kg}}$

Givens	*Solutions*
3. $E = 2.0 \times 10^3$ TW•h = 2.0×10^{15} W•h atomic mass of $^{235}_{92}$U = 235.043 924 u atomic mass of H = 1.007 825 u $m_n = 1.008\ 665$ u $Z = 92$ $N = 235 - 92 = 143$	$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } ^{235}_{92}\text{U}$ $\Delta m = 92(1.007\ 825\ \text{u}) + 143(1.008\ 665\ \text{u}) - 235.043\ 924\ \text{u} = 1.915\ 071\ \text{u}$ $E_{bind} = (1.915\ 071\ \text{u})\left(931.49\ \frac{\text{MeV}}{\text{u}}\right) = 1.7839 \times 10^3\ \text{MeV} = 1.7839 \times 10^9\ \text{eV}$ $E_{bind} = (1.7839 \times 10^9\ \text{eV})\left(1.60 \times 10^{-19}\ \frac{\text{J}}{\text{eV}}\right) = 2.85 \times 10^{-10}\ \text{J}$ $E = (2.0 \times 10^{15}\ \text{W•h})\left(\frac{3.60 \times 10^3\ \text{s}}{\text{h}}\right) = 7.2 \times 10^{18}\ \text{J}$ $n = \frac{E}{E_{bind}} = \frac{7.2 \times 10^{18}\ \text{J}}{2.85 \times 10^{-10}\ \text{J}} = 2.5 \times 10^{28}$ reactions $m_{tot} = (2.5 \times 10^{28})(235.043\ 924\ \text{u})\left(1.66 \times 10^{-27}\ \frac{\text{kg}}{\text{u}}\right) = \boxed{9.8 \times 10^3\ \text{kg}}$
4. $E = 2.1 \times 10^{19}$ J atomic mass of $^{12}_{6}$C = 12.000 000 u atomic mass of H = 1.007 825 u $m_n = 1.008\ 665$ u $Z = 6$ $N = 12 - 6 = 6$	$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } ^{12}_{6}\text{C}$ $\Delta m = 6(1.007\ 825\ \text{u}) + 6(1.008\ 665\ \text{u}) - 12.000\ 000\ \text{u}$ $\Delta m = 9.8940 \times 10^{-2}\ \text{u}$ $E_{bind} = (9.8940 \times 10^{-2}\ \text{u})\left(931.49\ \frac{\text{MeV}}{\text{u}}\right) = 92.162\ \text{MeV}$ $E_{bind} = (92.162 \times 10^6\ \text{eV})\left(1.60 \times 10^{-19}\ \frac{\text{J}}{\text{eV}}\right) = 1.47 \times 10^{-11}\ \text{J}$ $n = \frac{E}{E_{bind}} = \frac{2.1 \times 10^{19}\ \text{J}}{1.47 \times 10^{-11}\ \text{J}} = 1.4 \times 10^{30}$ reactions $m_{tot} = (1.4 \times 10^{30})(12.000\ 000\ \text{u})\left(1.66 \times 10^{-27}\ \frac{\text{kg}}{\text{u}}\right) = \boxed{2.8 \times 10^4\ \text{kg}}$
5. $P_{tot} = 3.9 \times 10^{26}$ J/s $Z = 2$ $N = 4 - 2 = 2$ atomic mass of $^{4}_{2}$He = 4.002 602 u atomic mass of H = 1.007 825 u m_n =1.008 665 u	$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } ^{4}_{2}\text{He}$ $\Delta m = (2)(1.007\ 825\ \text{u}) + (2)(1.008\ 665\ \text{u}) - 4.002\ 602\ \text{u}$ $\Delta m = 0.030\ 378\ \text{u}$ $E = (0.030\ 378\ \text{u})\left(931.49\ \frac{\text{MeV}}{\text{u}}\right)$ $E = 28.297\ \text{MeV}$ $\frac{n}{\Delta t} = \frac{P_{tot}}{E} = \frac{(3.9 \times 10^{26}\ \text{J/s})(1 \times 10^{-6}\ \text{MeV/eV})}{(1.60 \times 10^{-19}\ \text{J/eV})(28.297\ \text{MeV})}$ $\frac{n}{\Delta t} = \boxed{8.6 \times 10^{37}\ \text{reactions/s}}$
6. $P = 42$ MW $= 42 \times 10^6$ W atomic mass of $^{14}_{7}$N = 14.003 074 u atomic mass of H = 1.007 825 u $m_n = 1.008\ 665$ u $Z = 7$ $N = 14 - 7 = 7$ $\Delta t = 24$ h	$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } ^{14}_{7}\text{N}$ $\Delta m = 7(1.007\ 825\ \text{u}) + 7(1.008\ 665\ \text{u}) - 14.003\ 074\ \text{u}$ $\Delta m = 0.112\ 356\ \text{u}$ $E_{bind} = (0.112\ 356\ \text{u})\left(931.49\ \frac{\text{MeV}}{\text{u}}\right) = 104.66\ \text{MeV}$ $E_{bind} = (104.66 \times 10^6\ \text{eV})\left(1.60 \times 10^{-19}\ \frac{\text{J}}{\text{eV}}\right) = 1.67 \times 10^{-11}\ \text{J}$ $n = \frac{P\Delta t}{E_{bind}} = \frac{(42 \times 10^6\ \text{W})(24\ \text{h})(3600\ \text{s/h})}{1.67 \times 10^{-11}\ \text{J}} = 2.2 \times 10^{23}$ reactions $m_{tot} = (2.2 \times 10^{23})(14.003\ 074\ \text{u})\left(1.66 \times 10^{-27}\ \frac{\text{kg}}{\text{u}}\right) = \boxed{5.1 \times 10^{-3}\ \text{kg} = 5.1\ \text{g}}$

II

Givens	Solutions
7. $P = 3.84 \times 10^7$ W	$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } ^{12}_{6}\text{C}$
atomic mass of $^{12}_{6}$C = 12.000 000 u	$\Delta m = 6(1.007\ 825\ \text{u}) + 6(1.008\ 665\ \text{u}) - 12.000\ 000\ \text{u}$
atomic mass of H = 1.007 825 u	$\Delta m = 9.8940 \times 10^{-2}$ u
$m_n = 1.008\ 665$ u	$E_{bind} = (9.8940 \times 10^{-2}\ \text{u})\left(931.49 \times 10^6\ \dfrac{\text{eV}}{\text{u}}\right)\left(1.60 \times 10^{-19}\ \dfrac{\text{J}}{\text{eV}}\right)$
$Z = 6$	$E_{bind} = 1.47 \times 10^{-11}$ J
$N = 12 - 6 = 6$	$\dfrac{n}{\Delta t} = \dfrac{P}{E_{bind}} = \dfrac{3.84 \times 10^7\ \text{W}}{1.47 \times 10^{-11}\ \text{J}} = 2.61 \times 10^{18}$ reactions/s
	$\dfrac{m_{tot}}{\Delta t} = (2.61 \times 10^{18}\ \text{s}^{-1})(12.000\ 000\ \text{u})\left(1.66 \times 10^{-27}\ \dfrac{\text{kg}}{\text{u}}\right) = \boxed{5.20 \times 10^{-8}\ \text{kg/s}}$

Additional Practice B

1. $^{238}_{92}\text{U} + ^{1}_{0}n \rightarrow \text{X}$

$\text{X} \rightarrow ^{939}_{93}\text{Np} + ^{0}_{-1}e + \overline{\nu}$

$^{239}_{93}\text{Np} \rightarrow ^{239}_{94}\text{Pu} + ^{0}_{-1}e + \overline{\nu}$

mass number of X = 238 + 1 = 239

atomic number of X = 92 + 0 = 92 (uranium)

$\text{X} = ^{239}_{92}\text{U}$

The equations are as follows:

$$\boxed{\begin{array}{l} ^{238}_{92}\text{U} + ^{1}_{0}n \rightarrow ^{239}_{92}\text{U} \\ ^{239}_{92}\text{U} \rightarrow ^{939}_{93}\text{Np} + ^{0}_{-1}e + \overline{\nu} \\ ^{239}_{93}\text{Np} \rightarrow ^{239}_{94}\text{Pu} + ^{0}_{-1}e + \overline{\nu} \end{array}}$$

2. $\text{X} \rightarrow \text{Y} + ^{4}_{2}\text{He}$

$\text{Y} \rightarrow \text{Z} + ^{4}_{2}\text{He}$

$\text{Z} \rightarrow ^{212}_{83}\text{Bi} + ^{0}_{-1}e + \overline{\nu}$

mass number of Z = 212 + 0 = 212

atomic number of Z = 83 − 1 = 82 (lead)

$Z = ^{212}_{82}\text{Pb}$

mass number of Y = 212 + 4 = 216

atomic number of Y = 82 + 2 = 84 (polonium)

$\text{Y} = ^{216}_{84}\text{Po}$

mass number of X = 216 + 4 = 220

atomic number of X = 84 + 2 = 86 (radon)

$\text{X} = ^{220}_{86}\text{Rn}$

The equations are as follows:

$$\boxed{\begin{array}{l} ^{220}_{86}\text{Rn} \rightarrow ^{216}_{84}\text{Po} + ^{4}_{2}\text{He} \\ ^{216}_{84}\text{Po} \rightarrow ^{212}_{82}\text{Pb} + ^{4}_{2}\text{He} \\ ^{212}_{82}\text{Pb} \rightarrow ^{212}_{83}\text{Bi} + ^{0}_{-1}e + \overline{\nu} \end{array}}$$

3. $\text{X} \rightarrow ^{135}_{56}\text{Ba} + ^{0}_{-1}e + \overline{\nu}$

mass number of X = 135 + 0 = 135

atomic number of X = 56 + (−1) = 55 (cesium)

$\text{X} = ^{135}_{55}\text{Cs}$

$$\boxed{^{135}_{55}\text{Cs} \rightarrow ^{135}_{56}\text{Ba} + ^{0}_{-1}e + \overline{\nu}}$$