NAME ______________________ DATE ______________ CLASS ______________

Atomic Physics

Problem A

QUANTUM ENERGY

PROBLEM

Free-electron lasers can be used to produce a beam of light with variable wavelength. Because the laser can produce light with wavelengths as long as infrared waves or as short as X rays, its potential applications are much greater than for a laser that can produce light of only one wavelength. If such a laser produces photons of energies ranging from 1.034 eV to 620.6 eV, what are the minimum and the maximum wavelengths corresponding to these photons?

SOLUTION

Given:

$$E_1 = 1.034 \text{ eV}$$

$$= (1.034 \text{ eV})\left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}}\right) = 1.65 \times 10^{-19} \text{ J}$$

$$E_2 = 620.6 \text{ eV}$$

$$= (620.6 \text{ eV})\left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}}\right) = 9.93 \times 10^{-17} \text{ J}$$

$$h = 6.63 \times 10^{-34} \text{ J}\bullet\text{s}$$

$$c = 3.00 \times 10^{8} \text{ m/s}$$

Unknown: $\lambda_{min} = ?$ $\lambda_{max} = ?$

Use the equation for the energy of a quantum of light. Use the relationship between the frequency and wavelength of electromagnetic waves.

$$E = hf$$

$$f = \frac{c}{\lambda}$$

Substitute for f in the first equation, and rearrange to solve for wavelength.

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E}$$

Substitute values into the equation.

$$\lambda_{max} = \frac{(6.63 \times 10^{-34} \text{ J}\bullet\text{s})(3.00 \times 10^{8} \text{ m/s})}{(1.65 \times 10^{-19} \text{ J})}$$

$$\lambda_{max} = 1.21 \times 10^{-6} \text{ m}$$

$$\lambda_{max} = \boxed{1210 \text{ nm}}$$

$$\lambda_{min} = \frac{(6.63 \times 10^{-34} \text{ J}\bullet\text{s})(3.00 \times 10^{8} \text{ m/s})}{(9.93 \times 10^{-17} \text{ J})}$$

$$\lambda_{min} = 2.00 \times 10^{-9} \text{ m}$$

$$\lambda_{min} = \boxed{2.00 \text{ nm}}$$

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ADDITIONAL PRACTICE

1. In 1974, IBM researchers announced that X rays with energies of 1.29×10^{-15} J had been guided through a "light pipe" similar to optic fibers used for visible and near-infrared light. Calculate the wavelength of one of these X-ray photons.

2. Some strains of *Mycoplasma* are the smallest living organisms. The wavelength of a photon with 6.6×10^{-19} J of energy is equal to the length of one *Mycoplasma*. What is that wavelength?

3. Of the various types of light emitted by objects in space, the radio signals emitted by cold hydrogen atoms in regions of space that are located between stars are among the most common and important. These signals occur when the "spin" angular momentum of an electron in a hydrogen atom changes orientation with respect to the "spin" angular momentum of the atom's proton. The energy of this transition is equal to a fraction of an electron-volt, and the photon emitted has a very low frequency. Given that the energy of these radio signals is 5.92×10^{-6} eV, calculate the wavelength of the photons.

4. The camera with the fastest shutter speed in the world was built for research with high-power lasers and can expose individual frames of film with extremely high frequency. If the frequency is the same as that of a photon with 2.18×10^{-23} J of energy, calculate its magnitude.

5. Wireless "cable" television transmits images using radio-band photons with energies of around 1.85×10^{-23} J. Find the frequency of these photons.

6. In physics, the basic units of measurement are based on fundamental physical phenomena. For example, one second is defined by a certain transition in a cesium atom that has a frequency of *exactly* 9 192 631 770 s^{-1}. Find the energy in electron-volts of a photon that has this frequency. Use the unrounded values for Planck's constant ($h = 6.626\,0755 \times 10^{-34}$ J•s) and for the conversion factor between joules and electron volts (1 eV = $1.602\,117\,33 \times 10^{-19}$ J).

7. Consider an electromagnetic wave that has a wavelength equal to 92 cm, a length that corresponds to the longest ear of corn grown to date. What is the frequency corresponding to this wavelength? What is its photon energy? Express the answer in joules and in electron-volts.

8. The slowest machine in the world, built for testing stress corrosion, can be controlled to operate at speeds as low as 1.80×10^{-17} m/s. Find the distance traveled at this speed in 1.00 year. Calculate the energy of the photon with a wavelength equal to this distance.

Problem Workbook Solutions

Atomic Physics

Additional Practice A

Givens	*Solutions*
1. $E = 1.29 \times 10^{-15}$ J $C = 3.00 \times 10^{8}$ m/s $h = 6.63 \times 10^{-34}$ J•s	$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\ \text{J•s})(3.00 \times 10^{8}\ \text{m/s})}{1.29 \times 10^{-15}\ \text{J}}$ $\lambda = 1.54 \times 10^{-10}\ \text{m} = \boxed{0.154\ \text{nm}}$
2. $E = 6.6 \times 10^{-19}$ J $C = 3.00 \times 10^{8}$ m/s $h = 6.63 \times 10^{-34}$ J•s	$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\ \text{J•s})(3.00 \times 10^{8}\ \text{m/s})}{6.6 \times 10^{-19}\ \text{J}}$ $\lambda = \boxed{3.0 \times 10^{-7}\ \text{m}}$
3. $E = 5.92 \times 10^{-6}$ eV $C = 3.00 \times 10^{8}$ m/s $h = 6.63 \times 10^{-34}$ J•s	$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\ \text{J•s})(3.00 \times 10^{8}\ \text{m/s})}{(5.92 \times 10^{-6}\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})}$ $\lambda = \boxed{0.210\ \text{m}}$
4. $E = 2.18 \times 10^{-23}$ J $h = 6.63 \times 10^{-34}$ J•s	$E = hf$ $f = \frac{E}{h}$ $f = \frac{2.18 \times 10^{-23}\ \text{J}}{6.63 \times 10^{-34}\ \text{J•s}}$ $f = \boxed{3.29 \times 10^{10}\ \text{Hz}}$
5. $E = 1.85 \times 10^{-23}$ J $h = 6.63 \times 10^{-34}$ J•s	$f = \frac{E}{h} = \frac{1.85 \times 10^{-23}\ \text{J}}{6.63 \times 10^{-34}\ \text{J/s}} = \boxed{2.79 \times 10^{10}\ \text{Hz}}$
6. $f = 9\,192\,631\,770\ \text{s}^{-1}$ $h = 6.626\,0755 \times 10^{-34}$ J•s 1 eV $= 1.602\,117\,33 \times 10^{-19}$ J	$E = hf$ $E = \frac{(6.626\,0755 \times 10^{-34}\ \text{J•s})(9\,192\,631\,770\ \text{s}^{-1})}{1.602\,117\,33 \times 10^{-19}\ \text{J/eV}}$ $E = \boxed{3.801\,9108 \times 10^{-5}\ \text{eV}}$
7. $\lambda = 92\ \text{cm} = 92 \times 10^{-2}$ m $c = 3.00 \times 10^{8}$ m/s $h = 6.63 \times 10^{-34}$ J•s $h = 4.14 \times 10^{-15}$ eV•s	$f = \frac{c}{\lambda}$ $f = \frac{3.00 \times 10^{8}\ \text{m/s}}{92 \times 10^{-2}\ \text{m}}$ $f = \boxed{3.3 \times 10^{8}\ \text{Hz} = 330\ \text{MHz}}$ $E = hf$

II

Givens *Solutions*

$E = (6.63 \times 10^{-34}\ \text{J}\bullet\text{s})(3.3 \times 10^{8}\ \text{Hz})$

$E = \boxed{2.2 \times 10^{-25}\ \text{J}}$

$E = (4.14 \times 10^{-15}\ \text{eV}\bullet\text{s})(3.3 \times 10^{8}\ \text{Hz})$

$E = \boxed{1.4 \times 10^{-6}\ \text{eV}}$

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8. $v = 1.80 \times 10^{-17}$ m/s

$\Delta t = 1.00$ year

$\lambda = \Delta x$

$c = 3.00 \times 10^{8}$ m/s

$h = 6.63 \times 10^{-34}$ J•s

$\Delta x = v\Delta t$

$$\Delta x = (1.80 \times 10^{-17}\ \text{m/s})(1.00\ \text{year})\left(\frac{365.25\ \text{days}}{1\ \text{year}}\right)\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)$$

$\Delta x = \boxed{5.68 \times 10^{-10}\ \text{m}}$

$$E = hf = \frac{hc}{\lambda} = \frac{hc}{\Delta x}$$

$$E = \frac{(6.63 \times 10^{-34}\ \text{J}\bullet\text{s})(3.00 \times 10^{8}\ \text{m/s})}{5.68 \times 10^{-10}\ \text{m}}$$

$E = \boxed{3.50 \times 10^{-16}\ \text{J}}$

II

Additional Practice B

1. $hf_t = 4.5$ eV

$KE_{max} = 3.8$ eV

$h = 4.14 \times 10^{-15}$ eV•s

$$f = \frac{[KE_{max} + hf_t]}{h} = \frac{[3.8\ \text{eV} + 4.5\ \text{eV}]}{4.14 \times 10^{-15}\ \text{eV}\bullet\text{s}} = \boxed{2.0 \times 10^{15}\ \text{Hz}}$$

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2. $hf_t = 4.3$ eV

$KE_{max} = 3.2$ eV

$h = 4.14 \times 10^{-15}$ eV•s

$KE_{max} = hf - hf_t$

$$f = \frac{KE_{max} + hf_t}{h}$$

$$f = \frac{3.2\ \text{eV} + 4.3\ \text{eV}}{4.14 \times 10^{-15}\ \text{eV}\bullet\text{s}}$$

$f = \boxed{1.8 \times 10^{15}\ \text{Hz}}$

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3. $hf_{t,Cs} = 2.14$ eV

$hf_{t,Se} = 5.9$ eV

$h = 4.14 \times 10^{-15}$ eV•s

$c = 3.00 \times 10^{8}$ m/s

$KE_{max} = 0.0$ eV for both cases

a. $KE_{max} = hf - hf_t = 0.0\ \text{eV} = \dfrac{hc}{\lambda} - hf_t$

$$\lambda = \frac{hc}{hf_t}$$

$$\lambda_{Cs} = \frac{hc}{hf_{t,Cs}} = \frac{(4.14 \times 10^{-15}\ \text{eV}\bullet\text{s})(3.00 \times 10^{8}\ \text{m/s})}{2.14\ \text{eV}}$$

$\lambda_{Cs} = \boxed{5.80 \times 10^{-7}\ \text{m} = 5.80 \times 10^{2}\ \text{nm}}$

b. $$\lambda_{Se} = \frac{hc}{hf_{t,Se}} = \frac{(4.14 \times 10^{-15}\ \text{eV}\bullet\text{s})(3.00 \times 10^{8}\ \text{m/s})}{5.9\ \text{eV}}$$

$\lambda_{Se} = \boxed{2.1 \times 10^{-7}\ \text{m} = 2.1 \times 10^{2}\ \text{nm}}$