NAME ______________________ DATE ____________ CLASS ____________

Subatomic Physics

Problem B

NUCLEAR DECAY

PROBLEM

The most stable radioactive nuclide known is tellurium-128. It was discovered in 1924, and its radioactivity was proven in 1968. This isotope undergoes two-step beta decay. Write the equations that correspond to this reaction.

SOLUTION

Given: $^{128}_{52}\mathrm{Te} \rightarrow {}^{0}_{-1}e + \mathrm{X} + \overline{\nu}$

$\mathrm{X} \rightarrow {}^{0}_{-1}e + \mathrm{Z} + \overline{\nu}$

Unknown: the daughter elements X and Z

The mass numbers and atomic numbers on both sides of the expression must be the same so that both charge and mass are conserved during the course of this particular decay reaction.

$$\text{Mass number of X} = 128 - 0 = 128$$
$$\text{Atomic number of X} = 52 - (-1) = 53$$

The periodic table shows that the nucleus with an atomic number of 53 is iodine, I. Thus, the first step of the process is as follows:

$$^{128}_{52}\mathrm{Te} \rightarrow {}^{0}_{-1}e + {}^{128}_{53}\mathrm{I} + \overline{\nu}$$

A similar approach for the second beta decay reaction gives the following equation. Again, the emission of an electron does not change the mass number of the nucleus. It does, however, change the atomic number by 1.

$$\text{Mass number of Z} = 128 - 0 = 128$$
$$\text{Atomic number of Z} = 53 - (-1) = 54$$

The periodic table shows that the nucleus with an atomic number of 54 is xenon, Xe. Thus the next step of the process is as follows:

$$^{128}_{53}\mathrm{I} \rightarrow {}^{0}_{-1}e + {}^{128}_{54}\mathrm{Xe} + \overline{\nu}$$

The complete two-step reaction is described by the two balanced equations below.

$$\boxed{\begin{aligned} &^{128}_{52}\mathrm{Te} \rightarrow {}^{0}_{-1}e + {}^{128}_{53}\mathrm{I} + \overline{\nu} \\ &^{128}_{53}\mathrm{I} \rightarrow {}^{0}_{-1}e + {}^{128}_{54}\mathrm{Xe} + \overline{\nu} \end{aligned}}$$

ADDITIONAL PRACTICE

1. Standard nuclear fission reactors use $^{235}_{92}\mathrm{U}$ for fuel. However, the supply of this uranium isotope is limited. Its concentration in natural uranium-238 is low, and the cost of enrichment is high. A good alternative is the breeder reactor in which the following reaction sequence occurs: $^{238}_{92}\mathrm{U}$ captures a neutron, and the resulting isotope emits a $^{0}_{-1}e$ particle to form

$^{239}_{93}$Np. This nuclide emits a second $^{0}_{-1}e$ particle to form $^{239}_{94}$Pu, which is fissionable and can be used as an energy-producing material. Write balanced equations for each of the reactions described.

2. Radon has the highest density of any gas. Under normal conditions radon's density is about 10 kg/m^3. One of radon's isotopes undergoes two alpha decays and then one beta decay (β^-) to form $^{212}_{83}$Bi. Write the equations that correspond to these reaction steps.

3. Every element in the periodic table has isotopes, and cesium has the most: as of 1995, 37 isotopes of cesium had been identified. One of cesium's most stable isotopes undergoes beta decay (β^-) to form $^{135}_{56}$Ba. Write the equation describing this beta-decay reaction.

4. Fission is the process by which a heavy nucleus decomposes into two lighter nuclei and releases energy. Uranium-235 undergoes fission when it captures a neutron. Several neutrons are produced in addition to the two light daughter nuclei. Complete the following equations, which describe two types of uranium-235 fission reactions.

$$^{235}_{92}\mathrm{U} + ^{1}_{0}n \rightarrow ^{144}_{56}\mathrm{Ba} + ^{89}_{36}\mathrm{Kr} + \underline{\qquad}$$

$$^{235}_{92}\mathrm{U} + ^{1}_{0}n \rightarrow ^{140}_{54}\mathrm{Xe} + \underline{\qquad} + 2\,^{1}_{0}n$$

5. The maximum safe amount of radioactive thorium-228 in the air is 2.4×10^{-19} kg/m^3, which is equivalent to about half a kilogram distributed over the entire atmosphere. One reason for this substance's high toxicity is that it undergoes alpha decay in which gamma rays are produced as well. Write the equation corresponding to this reaction.

6. The 1930s were notable years for nuclear physics. In 1931, Robert Van de Graaff built an electrostatic generator that was capable of creating the high potential differences needed to accelerate charged particles. In 1932, Ernest O. Lawrence and M. Stanley Livingston built the first cyclotron. In the same year, Ernest Cockcroft and John Walton observed one of the first artificial nuclear reactions. Complete the following equation for the nuclear reaction observed by Cockcroft and Walton.

$$^{1}_{1}p + ^{7}_{3}\mathrm{Li} \rightarrow \underline{\qquad} + ^{4}_{2}\mathrm{He}$$

7. Among the naturally occurring elements, astatine is the least abundant, with less than 0.2 g present in Earth's entire crust. The isotope $^{217}_{85}$At accounts for only about 5×10^{-9} g of all astatine. However, this highly radioactive isotope contributes nothing to the natural abundance of astatine because when it is created, it immediately undergoes alpha decay. Write the equation for this decay reaction.

Givens / *Solutions*

7. $P = 3.84 \times 10^7$ W

atomic mass of ${}^{12}_{6}\mathrm{C}$ = 12.000 000 u

atomic mass of H = 1.007 825 u

$m_n = 1.008\ 665$ u

$Z = 6$

$N = 12 - 6 = 6$

$\Delta m = Z(\text{atomic mass of H}) + Nm_n - \text{atomic mass of } {}^{12}_{6}\mathrm{C}$

$\Delta m = 6(1.007\ 825\ \mathrm{u}) + 6(1.008\ 665\ \mathrm{u}) - 12.000\ 000\ \mathrm{u}$

$\Delta m = 9.8940 \times 10^{-2}\ \mathrm{u}$

$$E_{bind} = (9.8940 \times 10^{-2}\ \mathrm{u})\left(931.49 \times 10^6\ \frac{\mathrm{eV}}{\mathrm{u}}\right)\left(1.60 \times 10^{-19}\ \frac{\mathrm{J}}{\mathrm{eV}}\right)$$

$E_{bind} = 1.47 \times 10^{-11}\ \mathrm{J}$

$$\frac{n}{\Delta t} = \frac{P}{E_{bind}} = \frac{3.84 \times 10^7\ \mathrm{W}}{1.47 \times 10^{-11}\ \mathrm{J}} = 2.61 \times 10^{18}\ \text{reactions/s}$$

$$\frac{m_{tot}}{\Delta t} = (2.61 \times 10^{18}\ \mathrm{s}^{-1})(12.000\ 000\ \mathrm{u})\left(1.66 \times 10^{-27}\ \frac{\mathrm{kg}}{\mathrm{u}}\right) = \boxed{5.20 \times 10^{-8}\ \mathrm{kg/s}}$$

Additional Practice B

1. ${}^{238}_{92}\mathrm{U} + {}^{1}_{0}n \rightarrow \mathrm{X}$

$\mathrm{X} \rightarrow {}^{939}_{93}\mathrm{Np} + {}^{0}_{-1}e + \overline{\nu}$

${}^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + {}^{0}_{-1}e + \overline{\nu}$

mass number of X = 238 + 1 = 239

atomic number of X = 92 + 0 = 92 (uranium)

$\mathrm{X} = {}^{239}_{92}\mathrm{U}$

The equations are as follows:

$$\boxed{\begin{array}{l} {}^{238}_{92}\mathrm{U} + {}^{1}_{0}n \rightarrow {}^{239}_{92}\mathrm{U} \\ {}^{239}_{92}\mathrm{U} \rightarrow {}^{939}_{93}\mathrm{Np} + {}^{0}_{-1}e + \overline{\nu} \\ {}^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + {}^{0}_{-1}e + \overline{\nu} \end{array}}$$

2. $\mathrm{X} \rightarrow \mathrm{Y} + {}^{4}_{2}\mathrm{He}$

$\mathrm{Y} \rightarrow \mathrm{Z} + {}^{4}_{2}\mathrm{He}$

$\mathrm{Z} \rightarrow {}^{212}_{83}\mathrm{Bi} + {}^{0}_{-1}e + \overline{\nu}$

mass number of Z = 212 + 0 = 212

atomic number of Z = 83 − 1 = 82 (lead)

$Z = {}^{212}_{82}\mathrm{Pb}$

mass number of Y = 212 + 4 = 216

atomic number of Y = 82 + 2 = 84 (polonium)

$\mathrm{Y} = {}^{216}_{84}\mathrm{Po}$

mass number of X = 216 + 4 = 220

atomic number of X = 84 + 2 = 86 (radon)

$\mathrm{X} = {}^{220}_{86}\mathrm{Rn}$

The equations are as follows:

$$\boxed{\begin{array}{l} {}^{220}_{86}\mathrm{Rn} \rightarrow {}^{216}_{84}\mathrm{Po} + {}^{4}_{2}\mathrm{He} \\ {}^{216}_{84}\mathrm{Po} \rightarrow {}^{212}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He} \\ {}^{212}_{82}\mathrm{Pb} \rightarrow {}^{212}_{83}\mathrm{Bi} + {}^{0}_{-1}e + \overline{\nu} \end{array}}$$

3. $\mathrm{X} \rightarrow {}^{135}_{56}\mathrm{Ba} + {}^{0}_{-1}e + \overline{\nu}$

mass number of X = 135 + 0 = 135

atomic number of X = 56 + (−1) = 55 (cesium)

$\mathrm{X} = {}^{135}_{55}\mathrm{Cs}$

$$\boxed{{}^{135}_{55}\mathrm{Cs} \rightarrow {}^{135}_{56}\mathrm{Ba} + {}^{0}_{-1}e + \overline{\nu}}$$

Givens | *Solutions*

4. $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + X$

mass number of X = 235 + 1 − 144 − 89 = 3

atomic number of X = 92 + 0 − 56 − 36 = 0 (neutron)

$X = 3\,^{1}_{0}n$

$$\boxed{^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^{1}_{0}n}$$

$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + Y + 2\,^{1}_{0}n$

mass number of Y = 235 + 1 − 140 − 2 = 94

atomic number of Y = 92 + 0 − 54 − 0 = 38 (strontium)

$Y = ^{94}_{38}Sr$

$$\boxed{^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n}$$

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5. $^{228}_{90}Th \rightarrow X + ^{4}_{2}He + \gamma$

mass number of X = 228 − 4 = 224

atomic number X = 90 − 2 = 88 (radium)

$X = ^{224}_{88}Ra$

$$\boxed{^{228}_{90}Th \rightarrow ^{224}_{88}Ra + ^{4}_{2}He + \gamma}$$

II

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6. $^{1}_{1}p + ^{7}_{3}Li \rightarrow X + ^{4}_{2}He$

mass number of X = 1 + 7 − 4 = 4

atomic number of X = 1 + 3 − 2 = 2 (helium)

$X = ^{4}_{2}He$

$$\boxed{^{1}_{1}p + ^{7}_{3}Li \rightarrow ^{4}_{2}He + ^{4}_{2}He}$$

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7. $^{217}_{85}At \rightarrow X + ^{4}_{2}He$

mass number of X = 217 − 4 = 213

atomic number of X = 85 − 2 = 83 (bismuth)

$X = ^{213}_{83}Bi$

$$\boxed{^{217}_{85}At \rightarrow ^{213}_{83}Bi + ^{4}_{2}He}$$

Additional Practice C

1. $T_{1/2}$ = 26 min, 43.53 s

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(26\text{ min})(60\text{ s/min}) + 43.53\text{ s}}$$

$$\lambda = \boxed{4.32 \times 10^{-4}\text{ s}^{-1}}$$

5 times the run time = 5 half-lives

percent of sample remaining = $(0.5)^5(100)$

percent decayed = 100 − percent remaining = 100 − $(0.5)^5(100)$ = $\boxed{96.875\text{ percent}}$